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ANATORY ARITHMETIC

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AN

EXPLANATORY ARITHMETIC

DESIGNED AS A

SCHOOL TEXT-BOOK,

BY WHICH A COURSE OF READING MAY BE CARRIED OUT
INTELLIGENTLY AND QUICKLY, HAVING AS
DISTINCTIVE FEATURES—

EXPLANATORY BOOK-WORK

IN A SHORT AND SIMPLE FORM, WITH EXAMPLES CHARTED OUT,
MODEL FORMS OF STATINGS, &c.

MISCELLANEOUS EXAMPLES AT THE END OF EACH SECTION; AND WITH
THE QUESTIONS, REFERENCES TO FOREGOING BOOK-WORK,

SO THAT THE REASONS FOR RESULTS MAY BE AGAIN
AND AGAIN CONSIDERED.

BY

G. EASTCOTT SPICKERNELL,

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PREFACE.

IN this text-book,—that Pupils may be required to read and think for themselves, and not depend too much on oral instruction,—Book-work is given in a simple form, and in as few words as seem consistent with proper explanations.

Examples have been so classified and arranged, that, by repetition, a thorough knowledge may be acquired; and that Pupils may not find themselves face to face with questions, at which they must stop until a Tutor can give them help, references to explanatory Book-work are given with questions set as Miscellaneous Exercises.

It is suggested that Pupils should be required to make good use of the references.

G. E. S.

BURGOYNE HOUSE, SOUTHSEA.

TABLE OF CONTENTS.

List of Subjects. PAGE

Numeration Table 1

I. Numeration 2

II. Notation 3

III. Simple Addition 5

IV. „ Subtraction 6

V. „ Multiplication 8

VI. „ Division 11

VII. Reduction 19

VIII. Compound Addition 25

IX. „ Subtraction 28

X. „ Multiplication 30

XI. „ Division 33

XII. Measure of Right-angled Surfaces . . . 37

XIII. „ „ „ Solids . . . 41

XIV. Greatest Common Measure 43

XV. Least Common Multiple 45

XVI. Vulgar Fractions 46

XVII. Decimal „ 60

XVIII. Circulating Decimals 66

XIX. Practice 73

XX. Proportion 78

XXI. Rule of Three 79

XXII. The Unitary Method 85

XXIII. Compound Proportion 86

XXIV. Interest, Simple and Compound 91

XXV. Discount 95

List of Subjects. PAGE
XXVI. Insurance, &c. 97
XXVII. Stocks 98
XXVIII. Profit and Loss 102
XXIX. Proportional Parts 105
XXX. Square Root 106
XXI. Cube Root 108

Miscellaneous Questions, Subjects (I.—VI.) . pp. 18, 19
" " " (VII.—XI.) . pp. 36, 37
" " " (XII.—XVI.) . pp. 58, 59, 60
" " " (XVII.—XVIII.) pp. 71, 72
" " " (XIX.) . p. 77
" " " (XXI.—XXIII.) pp. 88, 89, 90
" " " (XXIV.—XXXI.) pp. 110, 111, 112

EXPLANATORY ARITHMETIC.

1. The figures 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, are used to express *nought or nothing*, and the numbers *one*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine*.

Numbers are expressed by one figure up to nine as 7 that is, seven units or single things.

Numbers are then reckoned in groups of ten each up to ninety-nine, and a second figure placed on the left of the units is used to give the number of such *groups of ten units each*, as 67, i.e. six groups of ten each and seven units.

Groups of a *hundred units each* are then formed, and a third figure is used to express them as 567, that is five groups of a hundred each, &c.

The reckoning is then carried on in successive groups, *each consisting of ten of the group last formed* (14*), and being expressed by an additional figure.

2. The following Table gives the names of the different groups in the order of their formation, commencing from units. The *name* of each group *tells* the *number of units* of which that group consists.

u. = units.
t. of u. = tens of units.
h. of u. = hundreds of units.
th. = thousands.
t. of th. = tens of thousands.
h. of th. = hundreds of thousands.
m. = millions.
t. of m. = tens of millions.
h. of m. = hundreds of millions.
th. of m. = thousands of millions.
t. of th of m. = tens of ths. of millions.
h. of th. of m. = hunds. of ths. of millions.
b. = billions.
t. of b. = tens of billions.
h. of b. = hundreds of billions.
th. of b. = thousands of billions.
t. of th. of b. = tens of ths. of billions.
h. of th. of b. = hunds. of ths. of billions.

3. It will be seen from the foregoing Table that there are groups of *tens* and *hundreds* of UNITS,

,, ,, ,, THOUSANDS,
,, ,, ,, MILLIONS,
,, ,, ,, THOUSANDS OF MILLIONS,
,, ,, ,, BILLIONS,
,, ,, ,, THOUSANDS OF BILLIONS,

and should the reckoning be further extended of TRILLIONS and THOUSANDS OF TRILLIONS,—hence, it has been found convenient to deal with figures in ***sets of three groups each,*** the right hand or lowest group of a set giving the name of that set, thus,

h t	h t	h t	h t	h t	h t
000	000	000	000	000	000
th (of b)	**b**	**th (of m)**	**m**	**th**	**u**

4. NUMERATION. TO EXPRESS IN WORDS A NUMBER WRITTEN IN FIGURES, mark off in sets of three figures each, beginning at units.

In each set, the figures give the number, and the right hand or lowest group of a set gives the name of that set.

25	112	345	678	943
b	**th (of m)**	**m**	**th**	**u**

Twenty-five *billions*, one hundred and twelve *thousand*(*s* of millions), three hundred and forty-five *millions*, six hundred and seventy-eight *thousand*(*s*) nine hundred and forty-three (*units*).

Ex. I. Express in words—

1. 5724158
2. 39254317
3. 849631287
4. 3742195164
5. 7285419632
6. 18742158763
7. 216425328
8. 352187654
9. 372516483
10. 456789123

5. If three figures are not required to express the number of units, thousands, millions, &c., then cyphers

(0) will be found in each set in the places in which no figures are required.

3	002	020	001	010
b	th (of m)	m	th	u

Three *billions*, two *thousand* and twenty *millions*, one *thousand* and ten (*units*).

Ex. II. Express in words—

1. 30005020
2. 3050002
3. 3001025
4. 147015015
5. 3016051160
6. 127007070700
7. 2007015003
8. 3300003300
9. 10007003305
10. 4019070707

6. NOTATION. TO WRITE IN FIGURES ANY NUMBER EXPRESSED IN WORDS, underline or *note the names of* the different *sets* of figures, as given by the lowest group of each set, and *write* down *in figures the numbers* which belong to each set.

Express in figures, one hundred and five *thousand*, two hundred and ten *millions*, six hundred and three *thousand*, two hundred and five.

105	210	603	205
th	m	th	u

Ex. III. Express in figures the following—

1. Two hundred and six thousand, three hundred and forty-nine millions, four hundred and eighteen thousand, two hundred and eighty-four.

2. Twenty-nine thousand, three hundred and six millions, two hundred and forty-two thousand, nine hundred and fifty-eight.

3. Eight millions, eight hundred thousand, and eighty-two.

4. One hundred and eighteen millions, and twenty-seven.

5. Three billions, four hundred and seven thousand, five hundred and two millions, ninety-six thousand and eight.

6. Six hundred and sixteen thousand, four hundred and ten millions, three hundred and three thousand, and ninety-seven.

7. Thirteen thousand and three millions, seven thousand and five.

8. Four millions, forty thousand and fourteen.

7. Should a number be so expressed that there occur more than 9 hundreds in any set, or more than 9 of any group, in all cases, a change must be made into the next higher group, before expressing the given number in figures.

	h t	h t	h t	h t
(Highest possible figures)	999	999	999	999
	th	m	th	u

Express in figures, *twelve* hundred and eighty-three **thousand**, *eleven* hundred and ninety-five (**units**). [14*].

1,283,000	expressed.	1	284	195
1,195	separately, making	m	th	u

Ex. IV. Express in figures—

1. Sixteen hundred and three millions, four hundred and eight thousand, fourteen hundred and forty-two.

2. Nine hundred and fifty millions, thirteen hundred and seven thousand, four hundred and ninety-five.

3. Eleven hundred millions, forty-eight thousand, nineteen hundred and ninety-one.

4. Seventeen thousand, seventeen hundred and seventeen.

5. One million, one thousand and eighteen hundred.

6. Twelve hundred and eighty-six millions, twelve hundred and eighty-six thousand, twelve hundred and eighty-six.

8. In dealing with a number, it is often necessary to use only a part of it at one time, and in such a case, the *last figure of the part used gives the name of that particular part,*

	t	h t
Thus in	15	479
	th	u

1 | 5479 using 1.... is 1 ten thousand, &c.
15 | 479 „ 15... „ 15 thousand, &c.
154 | 79 „ 154.. „ 154 hundreds, &c.
1547 | 9 „ 1547. „ 1547 tens, &c.
15479 „ 15479 „ 15479 (units.)

9. The following signs are used in Arithmetic:—

+ (*plus, more*) shows that the number before which it is placed, is to be added.

− (*minus, less*) shows that the number is to be subtracted.

× (*into*) signifies multiplication.

÷ (*by*) „ division.

= (*equal to*) „ equality. (i.e. that is).

ADDITION.

10. When numbers are added or taken together *like must be added to like*, i.e. units to units, tens to tens, hundreds to hundreds, and so forth. In any case in which the number obtained exceeds nine, and is therefore expressed by two or more figures, like to like must be the guide, both in *leaving behind* and *carrying forward.*

th h t u			
5244	12 units, i.e.	t 12 u	2 u. left with units; 1 t. added to the tens.
5713	14 tens, i.e.	h t 14	4 t. left with tens; 1 h. added to the hundreds.
3985	19 hundreds, i.e.	h 19 th	9 h. left with hundreds; 1 th. added to thousands.
14942			

Ex. V. Add together—

1. 425134	2. 453162	3. 123453	4. 217634
334322	522435	165432	542315
512443	121543	216534	135243
45314	614323	543212	424656

5. 987653	6. 7346852	7. 3125891	8. 5512076
276874	8248358	1467259	3869125
3126	9786	7232614	4321878
54957	4137937	2387589	5467153
692473	4955948	1314675	2886749
548	234567	4872137	3254355

9. 4325786	10. 2546318	11. 3456782	12. 2540863
5783574	4457893	5632147	5765712
6234125	6215449	3564825	4486358
4839654	3564563	4135364	5128776
2355786	2467712	7946827	6749315
3645375	5874369	5567386	5364728
7256418	3556642	4468619	3759482

h t	h t	h t	h t	h t
000	000	000	000	000
b	th	m	th	u

13. Add together, six hundred and eighty-four thousand three hundred and eighty-five, thirty-eight thousand two hundred and nine, fifteen thousand and fifteen, four hundred thousand five hundred and ten.

14. Add together, forty-seven thousand three hundred and sixteen, four hundred and seventy thousand three hundred and six, seven hundred and four thousand five hundred and fifteen, thirteen thousand thirteen hundred (7) and thirteen.

15. Add together, eight millions eight hundred and eight thousand eight hundred and eight, eighty millions eighty thousand and eighty, eight hundred and eighty-eight millions eight hundred and eighty-eight thousand and eighty-eight, forty-five thousand six hundred and two, one million one thousand and one.

16. Add together, three hundred and five millions and fifty-two, three hundred and five thousand and twenty-five, one million and eighty-seven, eighty-seven millions and one, thirteen millions and thirteen thousand, thirteen thousand and thirteen, nine hundred and nine millions, ninety-three thousand and thirty-nine.

SUBTRACTION.

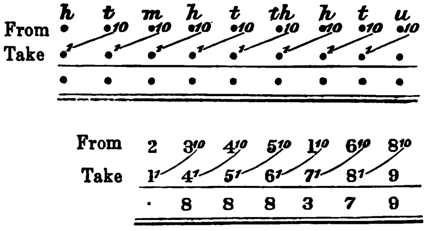
11. In taking one number from another, *like must be taken from like*, i.e., units from units, tens from tens, hundreds from hundreds, &c.

12. Here let it be observed that in the case of two numbers, their difference is not altered, if the same number be added to both, for example—

From	5	adding 3	8	adding 7	15	adding 9	24
Take	3	to each	6	to each	13	to each	22
Diff.	2		2		2		2

13. In subtracting one number from another, it is often necessary to make the same additions to both, but the additions to the less number or number-to-be-subtracted are different in form—though equal in value—from those made to the greater number.

14. Table showing the different forms in which additions must be made, when necessary.



*

10 u = 1 t
10 t = 1 h
10 h = 1 **th**
10 th = 1 t of th
10 t of th = 1 h of th
10 h of th = 1 **m**
&c.

Ex. VI. Examples in Subtraction.

1. 47689576 22337125	2. 4763576 1391847	3. 13571284 1588166	4. 5263453 4807284
5. 12304628 9476879	6. 13225078 8888469	7. 4235127 3748679	8. 11007050 3874683
9. 32546743 18768957	10. 13254621 3467528	11. 23546218 11768539	12. 31240050 17564685

	ht	ht	ht	ht
0	000	000	000	000
b	th	m	th	u

13. From two hundred and three thousand one hundred and fifty-four, *take* one hundred and eighty-five thousand, six hundred and seventy-eight.

14. From eight hundred and two thousand and sixty-five, *take* seventy-two thousand nine hundred and eighty-seven.

15. From thirteen millions four hundred and forty-two thousand five hundred and sixteen, *take* one million eight hundred and eighty thousand two hundred and seventy-six.

16. From one thousand two hundred and four millions, *take* nine hundred and nine millions and forty-five.

17. From eighteen hundred and four millions eleven hundred and three thousand (7), *take* nineteen millions nine hundred and nine thousand and nineteen.

Exercises on Subtraction.

15.

9	greater no.	or	minuend	9
5	less „	„	subtrahend	5
4	difference	„	remainder	4

The following results are to be noticed—

(*a*) $\text{min}^{d} - \text{subtr}^{d} = \text{rem}^{r}$ $9 - 5 = 4$

(*b*) $\text{min}^{d} - \text{rem}^{r} = \text{subtr}^{d}$ $9 - 4 = 5$

(*c*) $\text{subtr}^{d} + \text{rem}^{r} = \text{min}^{d}$ $5 + 4 = 9$

Ex. VII.

1. The subtrahend is 159, the remainder 18; find the minuend (*c*).
2. The greater of two numbers is 3179, the difference between them is 78; find the less (*b*).
3. The less of two numbers is 183, their difference is 18; find the greater (*c*).
4. 3126 is the difference between two numbers, of which 8937 is the greater; find the less (*b*).
5. The remainder is 140, and the subtrahend 259; find the minuend (*c*).
6. Of two numbers the greater is 354 more than the other, which is 176; find their difference (*a*).

MULTIPLICATION.

16. Multiplication gives the result of the *addition* of *several* of the *same number*, thus 8 and 8 and 8 and 8 added together make 32, and this result is entered in the Multiplication Table as "4 times 8 are 32," and since 4 and 4 and 4 and 4 and 4 and 4 and 4 and 4 make 32 (in multiplication table "8 times 4 are 32") we note that 4 times 8 and 8 times 4 give the same result.

17. The Multiplication Table gives the result of the addition of the same number taken two together, three together, four together, &c., up to twelve together.

			2	Twice 2 = 4
		2	2	Three times 2 = 6
	2	2	2	Four times 2 = 8
2	2	2	2	Five times 2 = 10
2	2	2	2	&c.
4	6	8	10	

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

18. Numbers worked together by multiplication are called *Factors;* the result is called the *Product.*

19. Multiply 4583 by 3.

In this, the work must be carried out in portions (8), and we must consider the number as consisting of the several parts, 4000 (4 thousands), 500 (5 hundreds), 80 (8 tens), and 3 units, and in multiplying we have to deal separately with units, tens, hundreds, and thousands. Should the number obtained in any one group exceed nine, as will be the case with the tens and hundreds and thousands in this example, *like to like* must be our guide both in *leaving behind* and *carrying forward.*

th h t u
4 5 8 3 multiplicand
3 multiplier
13749 product

3 times 3 units = 9 u
3 „ 8 tens = 24 t = $\overset{ht}{24}$
3 „ 5 hund^{s} = 15 h + 2 h = 17 h = $\underset{th}{\overset{h}{17}}$
3 „ 4 thous^{ds} = 12 th + 1 th = 13 th – $\underset{th}{\overset{t}{13}}$

Ex. VIII.

1. 345434 × 2	2. 367514 × 3	3. 235746 × 4
4. 738149 × 5	5. 214878 × 6	6. 236345 × 7
7. 182173 × 8	8. 573124 × 9	9. 935647 × 10
10. 254368 × 11	11. 643825 × 12	12. 475368 × 12

20. When the multiplier can be separated into factors, each factor being not more than 12, the factors may be used as multipliers, thus—

$$\begin{array}{rl}
47234 \times 28 = & 4 \times 7 \\
4 & \\
\hline
188936 = & 4 \text{ times} \\
7 & \\
\hline
1322552 = & 28 \text{ times}
\end{array}
\qquad\qquad
\left.\begin{array}{rl}
4 & \text{times} \\
4 & ,, \\
4 & ,, \\
4 & ,, \\
4 & ,, \\
4 & ,, \\
4 & ,, \\
\hline
28 & ,,
\end{array}\right| \begin{array}{l}\text{7 times} \\ \text{repeated.}\end{array}$$

Ex. IX.

1.	519432 × 16	2.	594356 × 25	3.	734695 × 45
4.	697128 × 36	5.	847319 × 96	6.	697821 × 81
7.	432157 × 121	8.	268544 × 132	9.	876543 × 144
10.	327456 × 63	11.	426738 × 56	12.	485279 × 77

21. Multiply 47133 by $\overset{\dagger}{3}\overset{*}{4}8$.

In this example, we have to find the result if the number 47133 be repeated 8 times, 40 times (4 times 10 times), and also 300 times (3 times 100 times, or 3 times 10 times 10 times).

$$\begin{array}{rl}
47133 & \\
8 & \\
\hline
377064 = & 8 \text{ times}
\end{array}
\qquad
\begin{array}{rl}
47133 & \\
10 & \\
\hline
471330 = & 10 \text{ times} \\
4^{*} & \\
\hline
1885320 = & 40 \text{ times} \\
{*} \quad &
\end{array}
\qquad
\begin{array}{rl}
47133 & \\
10 & \\
\hline
471330 = & 10 \text{ times} \\
10 & \\
\hline
4713300 = & 100 \text{ times} \\
3^{\dagger} & \\
\hline
14139900 = & 300 \text{ times} \\
\dagger \quad &
\end{array}$$

22. It is to be observed from the above, in which multiplication by 10 occurs three times, that the *figures* are *not altered*, *but* that their *positions* are *changed* by such multiplication by 10; and it is also to be remarked that when the *multiplying figure* is a number of *units*, as 8, the first figure of the product is *units*, when the *multiplying* *figure is tens*, as $\overset{*}{4}$ tens, *the first figure* OF ANY VALUE in the product *is tens*, when *hundreds* as 3[†] hundreds the *first figure* OF ANY VALUE *is hundreds;* hence the right place is apportioned to figures, when the first figure obtained in a product is placed under the multiplying figure.

$$
\begin{array}{r}
47133 \\
348 \\
\hline
.....4 \\
.....2 \\
.....9 \\
\hline
\end{array}
\qquad\qquad
\begin{array}{rcrl}
47133 & & & \\
348 & & & \\
\cline{1-1}
377064 & = & 8 & \text{times} \\
188532 & = & 40 & \text{,,} \\
141399 & = & 300 & \text{,,} \\
\cline{1-1}
16402284 & = & 348 & \text{,,}
\end{array}
$$

Ex. X.

1. 5196432 × 123
2. 525193 × 168
3. 478556 × 745
4. 926193 × 369
5. 235486 × 516
6. 470328 × 506
7. 558877 × 708
8. 467586 × 670
9. 514834 × 509
10. 434567 × 789
11. 473582 × 2478
12. 496532 × 3569

Ex. XI.

1. 3405006 × 1200
2. 4075060 × 370
3. 2578690 × 4900
4. 1235786 × 2079
5. 3102057 × 3080
6. 1400520 × 760
7. 1021005 × 1707
8. 3004157 × 1760
9. 4030507 × 870

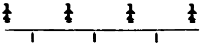
DIVISION.

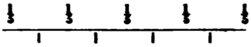
23. Division enables us to find how often one number occurs or is contained in another number.

The result could be obtained by repeated subtraction, but in most cases the work would be exceedingly long and tedious.

24. The Multiplication Table, in giving the result when the same number is repeated a certain number of times, tells how often a certain number must be repeated to make up a particular number, thus, if 8 repeated 7 times makes 56, 8 is contained in 56 7 times, and if 7 times 8 are 56, then 7 is contained in 56 8 times; hence the Multiplication Table is available also for division.

25. When a whole thing or integer is divided into a number of equal parts, the name of such parts is taken from the number of them; thus, in division into 4 equal parts, each part is called *one-fourth*, written $\frac{1}{4}$; into 5 equal parts, each part is called *one-fifth*, written $\frac{1}{5}$; and any other number of such parts would be written thus, three-fourths $\frac{3}{4}$, four-fifths $\frac{4}{5}$.





26. Since the number 24 taken as 8 + 8 + 8 is in that case divided into three equal parts, each part is one-third, that is, 8 is $\frac{1}{3}$ of 24, and hence the result obtained by division is two-fold.

$$3\overline{)\,24}\qquad\qquad 11\overline{)\,55}$$
$$\underline{}8\qquad\qquad\quad \underline{}5$$

(*a*) 8 = no. of times 3 occurs in 24
(*b*) 8 = one-third of 24.

(*a*) 5 = no. of times 11 occurs in 55.
(*b*) 5 = $\frac{1}{11}$ of 55.

27. Divide 7693 by 3.

```
   th     h     t     u
   7      6     9     3                    th  h  t  u
         16    19    13                  3)7   6  9  3
3) 6+1  15+1  18+1  12+1                   ------------
   ------------------------                 2   5  6  4
   2      5     6     4....1 u. undivided.  th  h  t  u-1
                                                      u
```

From the Multiplication Table we know that 6 9 12 15 18 21 24 27 30 33 36 are the numbers in which 3 occurs an exact number of times, and these will be our *guiding numbers.*

Commencing with the 7 thousands, guided by the numbers above, we deal with them in the two separate parts 6 *thousands* and 1 thousand.

Next, the 1 thousand (10 hundreds) [14*] associated with the next figure 6 hundreds, gives 16 hundreds, in this case guided by the Multiplication Table, we break up the number into 15 *hundreds* and 1 hundred.

The 1 hundred (10 tens) [14*] associated with the next figure 9 tens, makes up 19 tens, to be dealt with as 18 *tens* and 1 ten.

The 1 ten (10 units) [14*] with 3 units makes 13 units, i.e. 12 *units* and 1 unit; hence the result becomes

the third of 6 thousands = 2 thousands,
„ 15 hundreds = 5 hundreds,
„ 18 tens = 6 tens,
„ 12 units = 4 units,

and 1 unit remains untaken.

28. Divide 9786932 by 4.

The numbers formed by the repetition of 4; that is, 8 12 16 20 24 28 32 36 40 44 48 are the *guiding numbers.*

	m	**h**	**t**	**th**	**h**	**t**	**u**
	9	7	8	6	9	3	2
		17	18	26	29	13	12
4)	8+1	16+1	16+2	24+2	28+1	12+1	
	2	4	4	6	7	3	3

	m	**h**	**t**	**th**	**h**	**t**	**u**
4)	9	7	8	6	9	3	2
	2	4	4	6	7	3	3
	m	**h**	**t**	**th**	**t**	**h**	**u**

Ex. XII.

1. 615234 ÷ 2
2. 519317 ÷ 3
3. 823546 ÷ 4
4. 576493 ÷ 5
5. 394254 ÷ 6
6. 543756 ÷ 7
7. 428625 ÷ 8
8. 397609 ÷ 9
9. 248378 ÷ 11
10. 497865 ÷ 12
11. 465387 ÷ 7
12. 142896 ÷ 9

29. Factors are worked together by Multiplication, and are taken by Division out of numbers in which they occur.

4 × 3 × 5 = 60
factor 5) 60
factor 3) 12
factor 4

Ex. XIII.

1. Work together and then take apart the factors, 2, 3, 5, 8; 4, 7, 4; 12, 12, 12; 3, 9, 8, 7; 2, 3, 4, 5, 6; 8, 2, 11, 10.

2. Resolve into factors, 75, 432, 605, 1728, 270, 288, 105, 343, 640, 1760.

3. Express in factors, 176, 1000, 175, 112, 550.

---

* The *one* divisor equal to two or more divisors is found by multiplication.

63 | 7)
 | 9)

30. When the divisor is made up of two or more factors, each being not greater than 12, it is often convenient to divide by the factors.

Divide 3720 by 15, that is, find $\frac{1}{15}$th of the given number; also how often 15 occurs in it. **26*b*, 26*a*.**

(*a*)

$$15\left|\begin{array}{l}5\underline{)\,3720} = 15\text{ times the no. sought}\\ 3\underline{)\,744} = \ 3\text{ times } ,, \quad ,, \qquad \text{since } \tfrac{1}{5}\text{ of 15 times is 3 times.}\\ \quad 248 = \begin{cases}1\text{ time } ,, \quad ,, \qquad ,, \ \tfrac{1}{3}\text{ of } 3 \ ,, \text{ is 1 time.}\\ \text{being } \tfrac{1}{15}\text{ of 15 times}\end{cases}\end{array}\right.$$

(*b*)

$$15\left|\begin{array}{l}5\underline{)\,3720}\\ 3\underline{)\,744} = \quad \text{no. of times 5 occurs}\\ \quad 248 = \tfrac{1}{3}\text{ of } ,, \quad ,, \quad ,,\end{array}\right.$$

Now 15, being 5 three times over, occurs $\frac{1}{3}$rd of the no. of times 5 occurs.

(*c*) Divide 2148968 by 21 3 × 7.

$$21\left|\begin{array}{l}3\underline{)\,2148968}\searrow\\ 7\underline{)\,716322}-2\\ \qquad\qquad\ \searrow\\ \ \ 102331-5\end{array}\right|\ 17$$

the first line contains the No. of units in the number given for division.
„ second „ „ threes „ „ „
hence the remainders are 2 units and 5 threes = 17.

(*d*) Divide 4162999 by 112 4 × 7 × 4.

$$112\left|\begin{array}{ll}4\underline{)\,4162999}\searrow & \text{1st line contains number of units.}\\ 7\underline{)\,1040749}-3 & \text{2nd } ,, \quad ,, \quad ,, \quad \text{fours.}\\ \qquad\qquad\ \searrow & \\ 4\underline{)\,148678}-3 & \text{3rd } ,, \quad ,, \quad ,, \quad \text{twenty-eights.}\\ \qquad\qquad\ \searrow & \\ \ \ 37169-2 & \end{array}\right.$$

hence the remainders are 3 units + 3 fours + 2 twenty-eights = 3 + 12 + 56 = 71.

31. Form which will give the sum of any number of remainders.

$$\begin{array}{r}\text{1st rem.} =\\ \text{2nd rem.} \times \text{1st div.} =\\ \text{3rd rem.} \times \text{2nd div.} \times \text{1st div.} =\\ \text{4th rem.} \times \text{3rd div.} \times \text{2nd div.} \times \text{1st div.} =\\ \text{\&c.}\qquad\qquad\qquad\qquad \end{array}\ \underline{\qquad\qquad}$$

32. Divide 47896 by 34.

One method of Proof. *Rem*^{r} + *subtrahends* = *dividend.*

```
     t of
     thth  h  t  u    th  h  t  u                              Proof
3 4) 4  7  8  9  6   (1  4  0  8    3.) 4....(1408          )47896(
     3  4                   3  4        ..                  ──────
     ────            ────────────       ──                  34
     1  3  8 h.         5  6  3  2      13.                 ─
     1  3  6         4  2  2  4         ...                 ...
     ─────── t.               2  4      ───                 136
           ⌒         ────────────         2 | 9.            ───
           2  9  6 u. 4  7  8  9  6       .  ..               ...
           2  7  2                        ─────               272
           ───────                           ..               ───
              2  4 rem^r.                                      24 rem.
                                                            ═════
                                                            47896 div^d.
```

Since 34 is not contained in 4 tens of thousands, we must begin with 47 thousands (8), and it is to be observed that not more than 1 thousand is contained 34 times in 47 thousands; 1 thousand is then entered in the result as occurring 34 times in 34 thousands.

Having thus far dealt only with 34 thousands out of the given number, we have 13 thousands yet untouched; associating these 13 thousands (130 hundreds) [14*] with the next part of the given number—8 hundreds—we get 138 hundreds, and it will be found that 4 hundreds occurring 34 times, make up 136 hundreds.

Subtracting we have 2 hundreds (20 tens) [14*] and these with the next figure 9 tens, make up $\overset{\frown}{29}$ tens; but as not even 1 ten occurs 34 times in 29 tens, we change to units, 290 units, and these with the last figure 6 units make up 296 units, in which 8 units occurring 34 times make 272 units, and there remain 24 units over.

To find what number of thousands, hundreds, tens, and units repeated 34 times, will be likely to bring the result nearly up to 47 thousands, 138 hundreds, 29 tens, and 296 units respectively, it will be found sufficient to take only the highest figures of the divisor **3**4, and of the several dividends **4**7 **13**8 **2**9 **29**6, and find (middle form) what number repeated 3 times will almost make up 4 13 2 29, the results being 1, 4, 0, 9; the first three are correct; the last one 9 will be found too high, but will serve as a guide to the right number 8.

Divide 3987168 by 65.

```
   t of
    th                                                 Proof.
     ⌒
65) 3987168 (61341        6.) 39.....(61341          )3987168(
    390                       ...                    ---------
    ---                       ---                     390
      87 thousands              8.                    ---
      65                       ..                      ..
      --                       --                      65
      221 hundreds             22.                     --
      195                      ...                     ...
      ---                      ---                     195
       266 tens                 26.                    ---
       260                     ...                      ...
       ---                     ---                      260
         68 units                6.                     ---
         65                      ..                       ..
         --                      --                       65
          3                       .                       --
                                                           3 rem.
                                                     ---------
                                                      3987168
                         t of
                         th  h t
          Result  =      6 1 3 4 1
                         th      u
```

In the example above, it has to be considered what numbers, 6 times repeated, will almost make up 39 8 22 26 6, and will indicate the 65th part of 398 tens of thousands
„ „ 87 thousands
„ „ 221 hundreds
„ „ 266 tens
„ „ 68 units.

Ex. XIV. (using factors).

1. 2367546 ÷ 18
2. 2739456 ÷ 32
3. 7825870 ÷ 63
4. 6589438 ÷ 84
5. 5061987 ÷ 88
6. 42385716 ÷ 24
7. 3456780 ÷ 96
8. 3355748 ÷ 132
9. 2489788 ÷ 45
10. 4887965 ÷ 70
11. 2413256 ÷ 42
12. 2184163 ÷ 35

1. 47351628 ÷ 112
2. 55678932 ÷ 1728
3. 4238965 ÷ 112
4. 54987632 ÷ 144
5. 87695886 ÷ 112
6. 7643897 ÷ 1728
7. 24153129 ÷ 1728
8. 21003125 ÷ 147
9. 10051362 ÷ 125

Ex. XV.

1. 47569873 ÷ 23
2. 34278965 ÷ 43
3. 35486974 ÷ 37
4. 28975627 ÷ 46
5. 31068405 ÷ 57
6. 87986753 ÷ 237
7. 45887986 ÷ 376
8. 98996548 ÷ 487
9. 48878658 ÷ 633
10. 51200807 ÷ 738
11. 217896584378 ÷ 879
12. 135897688753 ÷ 789.

33. *Exercises on Multiplication and Division.*

9 Multiplicand (*a*) multd. × multr. = prod. $9 \times 4 = 36$
4 Multiplier (*b*) prod. ÷ multr. = multd. $36 \div 4 = 9$
36 Product (*c*) prod. ÷ multd. = multr. $36 \div 9 = 4$

Ex. XVI. (A).

1. The product is 1728, and the multiplier 12, find the multiplicand (*b*).
2. The product is 1563156, and the multiplicand 37218, find the multiplier (*c*).
3. The multiplier is 692, and the product 2399164, find the multiplicand (*b*)?
4. The multiplier is 73, and the multiplicand four times that number, find the product (*a*).
5. The product is 20736, and the multiplier 36, what is the multiplicand (*b*)?
6. The product is 47872, and the multiplicand 1408, find the multiplier (*c*).

Exercises on Division.

34. Firstly, in examples in which no remainder occurs.

div^{r} divid^{d} quot^{t}.
1 2) 1 3 2 (1 1 (*a*) divid^{d} ÷ div^{r} = quot^{t} $132 \div 12 = 11$.
1 3 2 (*b*) divid^{d} ÷ quot^{t} = div^{r} $132 \div 11 = 12$.
——— (*c*) div^{r} × quot^{t} = divid^{d} $12 \times 11 = 132$.

that is, of the three, Divisor, Dividend, Quotient, any two being known, the third can be found.

Secondly, in examples in which a remainder occurs, as

div^{r} divid^{d} quot^{t}. The dividend less the remainder ($135-3=132$)
1 2) 1 3 5 (1 1 will give a dividend that contains the divisor an
1 3 2 exact number of times, and then
——— (*a*) as above.
3 rem. (*b*) as above.
but (*c*) div^{r} × quot^{t} + rem^{r} = divid^{d} $12 \times 11 + 3 = 135$.

Ex. XVI. (B).

1. The dividend is 1728, and the quotient 12, find the divisor (*b*).
2. The divisor is 42, the quotient 9036, find the dividend (*c*).
3. The quotient is 1793, and the divisor 54, find the dividend (*c*).
4. The dividend is 100000, and the quotient 250, find the divisor (*b*).
5. The divisor is 42, the quotient 9036, and the remainder 31, find the dividend (second case *c*).
6. The quotient is 3472, the divisor 24, and the remainder 1, find the dividend (second case *c*).

MISCELLANEOUS QUESTIONS (I.–XVI.).

1. Express in words, 398476. 4.

2. Express in words, 40007. 4.

3. Express in figures, one hundred and sixty-six thousand two hundred and eighty-four. 6.

4. Express in figures, ten thousand and forty-nine. 6.

5. Add together, 176134, 54873, 49 and 1527. 10.

6. From 1247853 take 751287. 13, 14.

7. Multiply 8357162 by 49. 20.

8. Multiply 8357162 by 49, using factors. 20.

9. Divide 123471652 by 28 32.

10. Divide 123471652 by 28, using factors. 30*c*.

11. Express in words, 218513127. 4.

12. Express in words, 50050500. 4.

13. Express in figures, six millions six hundred and seventy-two thousand five hundred and eighty-four. 6.

14. Express in figures, three millions four thousand and four. 6.

15. Add together 81354, 2715, 478, 96 and 9. 10.

16. From 123456 take 98765. 14.

17. Multiply 421387 by 144. 21.

18. Multiply 421387 by 144, using three factors. 29, 20.

19. Divide 235127 by 1728. 32.

20. Divide 235127 by 1728, using factors. 29, 30*d*.

21. Express in words, 123456789. 4.

22. Express in words, 20001001. 4.

23. Express in figures, one million and twelve thousands. 6.

24. Add together 71846, 1527139, 42865, 2398, 398, 87 and 1. 10.

25. Divide 172158 by 16, using factors. 29, 30.

26. Divide 2396137 by 28, using factors. 29, 30.

27. Divide 1488175 by 27, using factors. 29, 30.

28. Divide 14812756 by 144, using factors. 29, 30.

29. Divide 31268475 by 121, using factors. 29, 30.

30. Divide 3127864 by 21, using factors. 29, 30.

31. Express in words, 15176213512116. 4.

32. Express in words, 1001100010100. 4.

33. Express in figures, five hundred and two thousand and eighty-four millions six thousand three hundred and nine. 6.

34. Express in figures, twelve hundred and twelve thousand twelve hundred and twelve. 6.

35. Add together 176134, 2105738, 463, 2177 and 49, and give the sum in words. 10, 4.

36. From one million take eight thousand three hundred and seventy-two, and express the remainder in words. 6, 14, 4.

37. Multiply 17853462 by 9, and divide the product by 7. 19, 27.

38. Divide 413286593 by 1728, using factors. 29, 30*d.*

39. Divide 31456289 by 20. 29, 30.

40. Divide 413286593 by 87984. 32.

41. The greater of two numbers is 4783, and their difference is 500, find the less. 15*b.*

42. The less of two numbers is 2138, and their difference is 199, find the greater. 15*c.*

43. The subtrahend is 2718, and the remainder 49, find the minuend. 15*c.*

44. The minuend is 497, and the remainder 149, find the subtrahend. 15*b.*

45. The product is 173367270, and the multiplier 234915, find the multiplicand. 33*b.*

46. The product is 918588, and the multiplicand 76549, find the multiplier. 33*c.*

47. The divisor is 7152, and the quotient 1386, find the dividend. 34*c.*

48. The dividend is 47872, and the quotient 1408, find the divisor. 34*b.*

49. The quotient is 182, the remainder 3653, and the divisor 3900, find the dividend. 34*c*, second case.

50. The dividend is 713417, the remainder 3617, and the quotient 182, find the divisor. 34*b*, second case.

REDUCTION.

35. Numbers, considered without reference to any particular thing, are called abstract numbers, as 17, 18.

36. Concrete numbers are those used in connection with particular things, as 17 shillings, 15 tons.

37. Reduction is the name given to the method of finding an *equivalent number* of things—greater or less; as the number of pounds of equal value to 100 shillings; or the number of ounces of equal weight to 14 cwt.; or the number of inches of equal length to 4 miles.

Table of Money.

4 farthings (*f*) = 1 penny (*d*)   1 florin = 2 shillings.
12 pence = 1 shilling (*s*)   1 crown = 5 „
20 shillings = 1 pound £ or sovereign   1 guinea = 21 „

¼, ½, ¾, express one, two, and three farthings.

38. The table of money—as tables of weight, length, surface, &c.—indicates by inspection, how equivalent numbers can be obtained, for example: 4 farthings = 1 penny, i.e. the *number* (4) of farthings is *four times* the *number* (1) of pence; hence, multiply any number of pence by four to get a number four times as great, and that will be the number of farthings; also, since the *number* (1) of pence is *one-fourth* of the *number* (4) of farthings, divide any number of farthings by four to get one fourth of that number, as the equivalent number of pence, hence

No. of far^s^. 4 times No. of *d.*	No. of *d.* ¼ of No. of far^s^.
No. of *d.* 12 " " *s.*	No. of *s.* $\frac{1}{12}$ of " *d.*
No. of *s.* 20 " " £.	No. of £ $\frac{1}{20}$ of " *s.*

N.B.—Pupils should be exercised in reading off the Tables according to the foregoing method.

4) No. of farthings.
12) No. of *d.* being ¼ No. of farthings.
20) No. of *s.* " $\frac{1}{12}$ " *d.*
No. of £. " $\frac{1}{20}$ " *s.*

No. of £
20
No. of *s.* being 20 times No. of £.
12
No. of *d.* being 12 times No. of *s.*

	£	*s.*	*d.*	
Reduce	18	4	8¾	to farthings.
	20			
	364			4*s.* are here added.
	12			
	4376			8*d.* " "
	4			
	17507			3 far. " "

Ex. XVII. Reduce

1. £418 to farthings; 158327 farthings to pounds.
2. £763 to pence; 81754 pence to pounds.
3. £18 17*s.* to pence; £16 4*s.* 8½*d.* to farthings.
4. £27 16*s.* 10½*d.* to halfpence; 418 shillings to halfpence.
5. £48 10*s.* 6*d.* to sixpences; 236 crowns to shillings and pence.

6. 128573 half crowns to halfpence; 218456 halfpence to guineas.
7. 2345678 farthings to florins; 2345678 halfpence to half crowns.
8. 1516829 farthings to guineas; 4782365 guin. to shil. and pounds.
9. 482765 crowns to farthings; 121748 farthings to crowns.

39. Avoirdupois Weight (used for Common Goods).

16 drs. = 1 oz. 16 oz. = 1 lb. 28 lb. = 1 qr. 4 qrs. = 1 cwt. 20 cwt. = 1 ton.
also 14 lbs. = 1 stone. 2 stones = 1 qr. 112 lbs. = 1 cwt.

Ex. XVIII. Reduce

1. 17 cwt. to lbs.; 117 cwt. to ounces.
2. 18 lbs. to drs.; 108990 drams to cwt.
3. 2 cwt. 1 qr. 7 lbs. to oz; 4 tons 14 cwt. 2 qrs. to lbs.
4. 4765329 oz. to tons; 15 cwt. 1 qr. 0 lbs. 3 oz. to drs.
5. 12345678 drs. to qrs.; 227653 qrs. to tons.
6. 3 tons 3 cwt. 1 qr. 9 lbs. to oz.; 14 tons 3 cwt. 3 qrs. 15 lbs. to drs.
7. 11011011 oz. to cwt.; 12401015 drs. to tons.
8. 147 cwt. 1 qr. 27 lbs. to oz.; 34 cwt. 2 qrs. 18 lbs. 15 oz. to drs.
9. 12345678 oz. to cwt.; 14 tons 1 cwt. 2 qrs. 17 lbs. to lbs.

40. The following may be noted: 1 qr. = 28 lbs.; 2 qrs. = 56 lbs.; 3 qrs. = 84 lbs.; 1 cwt. = 112 lbs.; i.e. (100 lbs. + 12 lbs.), and the number of pounds may be found thus—

58 cwt. 2 qrs. 17 lbs. =	5800	100 × 58	4 cwt. 3 qrs. 16 lbs. =	400	100 × 4
	696	12 × 58		48	12 × 4
	56	= 2 qrs.		84	= 3 qrs.
	17			16	
	6569 lbs.			548 lbs.	

Ex. XIX. Reduce

1. 7 cwt. 2 qrs. 15 lbs. to lbs.
2. 8 cwt. 3 qrs. 5 lbs. to lbs.
3. 9 cwt. 1 qr. 18 lbs. „
4. 6 cwt. 2 qrs. 19 lbs. „
5. 17 cwt. 1 qr. 13 lbs. „
6. 18 cwt. 1 qr. 15 lbs. „
7. 115 cwt. 2 qrs. 7 lbs. „
8. 27 cwt. 3 qrs. 9 lbs. „

41. Troy Weight (Gold, Silver, Jewellery).

24 grs. = 1 dwt. 20 dwts. = 1 oz. 12 oz. = 1 lb.

Ex. XX. Reduce

1. 7 oz. 14 dwts. 21 grs. to grs.; 3 lbs. 4 oz. 7 grs. to grs.
2. 1461325 grs. to lbs.; 4781356 dwts. to oz.
3. 4 oz. 6 grs. to grs.; 51632786 grs. to lbs.
4. 1235168 grs. to oz.; 1217651 dwts. to lbs.

42. Apothecaries' Weight (Medical Prescriptions).

20 grains = 1 scruple. 3 scr. = 1 dram. 8 drs. = 1 oz. 12 oz. = 1 lb.

Ex. XXI. Reduce

1. **12 lbs. 1 oz. 2 scr. to grs.; 47138546 grs. to lbs. Apoth.**
2. **13526184 grs. to lbs.; 3 lbs. 1 scr. 1 gr. to grs.**
3. **4 oz. 2 scr. 18 grs. to grs.; 1421685 scr. to lbs.**

43. Length (Measure of a line).

12 in. = 1 foot. 3 ft. = 1 yd. 5½ yds. i.e. 11 half yds. = 1 pole. 40 po. = 1 fur. 8 fur. = 1 mile. 3 miles = 1 league. 220 yds. = 1 fur. 1760 yds. = 1 mile.

Reduce 3 m. 2 fur. 9 po. 3 yds. 1 ft. 6 in. to inches.

8	**† ***
26 fur.	**2 fur. added.**
40	
1409 po.	**9 po. „**
11	
2)11545 half yds.	**6 half yds. (3 yds.) added.**
5772 yds.	**1 half yd. = 1 ft. 6 in.**
3	**† ***
17318 ft.	**1 ft. + 1 ft. added.†**
12	
207828 in.	**6 in. + 6 in. „ ***

In 47125683 inches, how many miles?

12)47125683	**in.**
3)3927140	**ft. 3 in.**
1309046	**yds. 2 ft.**
2	
11)2618092	**half yds.**
40)238008	**po. 4 half yards = 2 yds.**
8)5950	**fur. 8 po.**
743	**m. 6 fur. 8 po. 2 yds. 2 ft. 3 in.**

Ex. XXII. Reduce

1. **4 m. 1 f. 5 p. 1 yd. 2 ft. 7 in. to inches; 51846379 in. to miles.**
2. **4 fur. 8 p. 4 yds. 3 ft. to inches; 248763 yds. to poles.**
3. **15 p. 1 yd. 1 ft. 7 in. to feet; 5132456 in. to poles.**
4. **2 leag. 4 m. 4 fur. 1 ft. to inches; 1718654 yds. to miles.**
5. **2 m. 500 yds. 3 in. to inches; 5 fur. 181 yds. 2 ft. to feet.**

6. 10000000 in. to miles; 3 leag. 2 m. 1 fur. to yards.
7. 171 miles to inches; 24716395 ft. to leagues.
8. 11 m. 1 fur. to feet; 8 p. 2 yds. 2 ft. 2 in. to feet.

44. Surface (Measurement of Surface by squares).

144 sq. in. = 1 sq. ft. 9 sq. ft. = 1 sq. yrd. 121 quar. yds. (30¼ sq. yds.) = 1 sq. pole or perch. 40 per. = 1 rood. 4 r. = 1 acre. 640 ac. = 1 sq. m. 4840 sq. yds. = 1 acre.

```
Reduce  1 a.  1 r.  3 per.  17 sq. yds.  2 sq. ft.  8 sq. in. to sq. inches.
        4                                †          *
       ---
        5 r.          1 rood added.
        40
       ---
       203 per.       3 per. added.
       121
       ---
       203
      406
     203                                        | 4) 9 ft.  0 in. = 1 sq. yd.
        68 qr. yds. i.e. 17 yds. added.         |   2 ft. 36 in. = ¼ sq. yd.
   4) 24631 qr. yds.                            |         3
      6157 yds.  3 qr. yrds = ..........  6 ft. 108 in. = ¾ sq. yd.
         9                                 †       *
     -----
     55421            2 ft. + 6 ft. added.†
       144
    ------
    221684
   221684
   55421
         8            8 in. + 108 in. added.*
       108
   -------
   7980740 sq. inches.

     Reduce  142758 sq. yds. to perches.
                  4
         | 11) 571032 qr. yds.
     121 |   11) 51911 − 1 | 23 qr. yds. = 5¾ yds. = 5 yds. 6 ft.
         |        4719 − 2 |            108 in. as above.
```

Ex. XXIII. Reduce

1. 2 a. 2 r. 4 per. 19 sq. yds. 2 sq. ft. 5 sq. in. to sq. inches.
2. 17 a. 3 r. 15 per. 21 sq. yds. 5 sq. ft. 7 sq. in. to sq. inches.
3. 4 a. 3 r. 18 p. 21¼ sq. yds. 2 sq. ft. 41 sq. in. to sq. inches.
4. In 41627853 sq. yds. how many perches?

5. In 2186327 sq. yds. how many acres?
6. In 121753684 sq. in. how many perches?
7. In 41250713 sq. ft. how many roods?
8. In 3 a. 1 r. 35 per. 18¼ yds. 1 sq. ft. 17 sq. in. how many inches.

45. Solidity (Measurement of solid bodies by cubes).

1728 cub. in. = 1 cub. ft. 27 cub. ft. = 1 cub. yd.

Ex. XXIV. Reduce

1. 15 cub. yds. to feet; 17 cub. yds. 25 cub. ft. to inches.
2. 18796532 cub. in. to yds.; 12 cub. ft. 1727 cub. in. to inches.
3. 14 cub. yds. 26 cub. ft. 1000 c. in. to in.; 10101010 cub. in. to yds.
4. 23456789 cub. in. to feet; 5 cub. yds. 5 ft. 15 in. to inches.

46. Capacity (Measurement of Liquids and Dry Goods).

4 gills = 1 pint. 2 pints = 1 quart. 4 quarts = 1 gallon. 2 galls. = 1 peck.
4 pecks = 1 bushel. 8 bush. = 1 quarter. 3 bush. = 1 sack.
36 galls. = 1 barrel (beer). 54 galls. = 1 hogshead (beer). 63 galls. =
1 hhd. (wine). 2 hhd. = 1 pipe (wine).

Ex. XXV. Reduce

1. 317 galls. to pints; 171483 pts. to gallons.
2. 14 galls. 1 qt. to gills; 19 qrs. to gallons.
3. 183476 pts. to gallons; 1512784 gills to gallons.
4. 2 qrs. 7 b. 5 pks. to gallons; 8 loads to gallons.

47. Time.

60 sec. = 1 minute. 60 min. = 1 hour. 24 hrs. = 1 day. 7 dys. = 1 week.
4 wks. = 1 mnth. (lunar). 52 wks. = 1 yr. 365 dys. = 1 yr. 366 dys. = 1 leap yr.

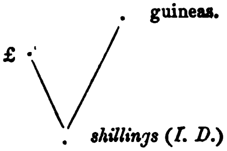
Ex. XXVI. Reduce

1. 37 yrs. to days; 4 yrs. 259 days to hours.
2. 14271586 min. to days; 46513278 secs. to weeks.
3. 7 d. 13 h. 7 m. 8 secs. to seconds; 10000000 secs. to days.
4. 12178564 hrs. to years; 1 yr. 15 dys. 4 hrs. to seconds.

48. In Reduction, *direct* changes that can be made (by Multiplication and Division), are shown by the several statements in the Tables of Money, Weight, Length, &c. (38), but it is not always possible at once to find an equivalent number of some other denomination, and the result has then to be obtained through an *Intermediate Denomination;* thus, to find the number of guineas equal to

£87 3*s*., it will be necessary first to find the number of shillings, and from that result, the number of guineas.

Reduce £87 3*s*. to guineas.
20
(29) 21 | 3) 1743 *s*.
7) 581
83 guineas.



Ex. XXVII. Reduce

1. 427 guin. to pounds, *I. D. shillings;* £279 to half guin., *I. D. sixp.*
2. 235 half crs. to guin., *I. D. sixpences;* 760 half guin. to half crs., *I. D. sixpences.*
3. 251 crs. to half guin., *I. D. sixpences;* 273 guin. to crs., *I. D. shill.*
4. 185734 fourpenny pieces to crs., *I. D. shillings;* 2500 half crs. to fourpenny pieces. *I. D. pence.*
5. £18 17*s*. 6*d*. to crs., *I. D. shillings;* £27 6*s*. 4*d*. to sixp., *I. D. pence.*
6. 3128 half guin. to fourpenny pieces, *I. D. pence;* £1001 to seven shilling pieces, *I. D. shillings.*
7. 219 ells to yds., *I. D. qrs.;* 500 fathoms to yds.., *I. D. feet.*

1 lb. Av. = 7000 grs. Troy. 1 lb. Troy = 5760 grs. Troy.

1. Reduce 1 cwt. 4 lbs. Av. to lbs. Troy, *I. D. grs. Troy;* 15 lbs. Av. to lbs. Troy. *I. D. grs. Troy.*
2. 114 lbs. Troy to lbs. Av., *I. D. grs. Troy;* 30 lbs. Troy to lbs. Av.. *I. D. grs. Troy.*

COMPOUND ADDITION, *i. e.* ADDITION OF CONCRETE NUMBERS.

49. Add like to like, as farthings to farthings, pence to pence, cwt. to cwt., and whenever a change (Reduction) can be made to the next higher denomination, it must be made, and "like to like" must guide both in leaving behind and carrying forward.

£	*s*.	*d*.
14	5	8
8	7	11
4	13	7
5	15	6
2	14	8
35	17	4

No. of *s*. = $\frac{1}{12}$ No. of *d*. 38.

12) 40*d*.
3*s*. 4*d*.

No. of £ = $\frac{1}{20}$ No. of *s*. 38.

20 | 10) 57*s*.
2) 5 − 7 | 17*s*. 30*c*.
2 − 1

dividing by 10 is shortly written thus 2,0) 5,7 leaving as in the form above, in which the division is given fully, 5 to be divided by 2.

Ex. XXVIII.

	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
1.	4	14	8	2.	18	15	8	3.	46	5	8¼	4.	28	18	7½
	1	11	9		23	17	10		23	7	6¾		35	16	2½
	3	13	4		91	11	11		29	15	7¼		49	18	6½
	2	5	7		28	16	6		37	12	9¾		18	4	5½

5.	17	15	6¼	6.	12	14	8½	7.	18	18	6	8.	77	14	6½
	36	8	9		33	11	1¾		17	16	8¾		35	5	10
	17	10	8		13	14	3½		90	8	9		21	18	8
	43	15	4¾		25	4	0		12	15	6¾		54	13	7¼

9.	14	6	9½	10.	63	17	7½	11.	25	15	7½	12.	17	19	5½
	21	11	8½		41	7	8½		15	14	2¾		14	5	8¾
	13	18	5½		35	13	9½		11	11	0		45	17	6½
	24	5	9½		14	14	8½		29	17	6¾		3	11	9

	lbs.	oz.	drs.		qrs.	lbs.	oz.		cwt.	qrs.	lbs.		qrs.	lbs.	oz.
13.	3	4	15	14.	3	27	14	15.	18	2	25	16.	13	27	8
	13	1	8		1	12	13		17	1	19		5	18	7
	32	13	15		0	12	13		15	3	18		25	17	5
	3	14	7		3	11	15		9	2	25		36	9	15

	qrs.	lbs.	oz.	drs.		cwt.	qrs.	lbs.	oz.		tons	cwt.	qrs.	lbs.	oz.
17.	2	14	13	12	18.	27	3	14	14	19.	2	15	1	9	3
	3	6	10	7		32	1	12	13		2	3	0	15	9
	1	27	12	5		29	0	14	13		13	19	2	25	8
	2	18	14	14		33	1	13	4		22	17	2	13	6

	oz.	dwts.	grs.		lbs.	oz.	dwts.		oz.	dwts.	grs.		lbs.	oz.	dwts.
20.	9	13	23	21.	22	7	15	22.	7	16	21	23.	25	8	13
	4	18	19		8	8	19		11	5	13		36	4	16
	6	6	15		36	4	4		4	15	20		52	10	9
	8	17	3		14	9	13		11	18	6		44	11	7

	lbs.	oz.	dwts.	grs.		lbs.	oz.	dwts.	grs.		lbs.	oz.	dwts.	grs.
24.	12	4	12	16	25.	35	2	5	13	26.	28	1	16	23
	24	8	18	12		27	8	13	21		5	9	1	23
	46	9	17	19		41	9	18	11		17	11	8	13
	31	4	11	17		2	3	13	21		23	6	9	15

	drs.	sc.	grs.		oz.	drs.	sc.		drs.	sc.	grs.		oz.	drs.	sc.
27.	4	2	15	28.	15	7	2	29.	7	1	18	30.	12	6	2
	7	1	13		4	3	2		4	1	19		10	5	2
	3	2	17		10	5	1		11	2	12		5	2	1
	6	0	11		9	4	1		9	1	14		11	6	2

	yds.	ft.	in.
31.	13	1	11
	23	2	9
	9	0	2
	14	1	5

	fur.	po.	yds.
32.	6	30	3½
	3	19	2½
	8	29	4
	4	34	5

	m.	fur.	yds.
33.	4	7	123
	2	4	131
	8	6	213
	3	5	27

	lea.	m.	fur.
34.	4	0	5
	8	1	7
	1	0	7
	9	2	7

	fur.	p.	yds.
35.	4	34	3½
	6	22	4½
	2	14	2½
	5	27	4

	p.	yds.	ft.
36.	28	4½	1
	36	4½	2
	42	3½	1
	14	4	2

	yds.	ft.	in.
37.	4	1	11
	9	2	9
	5	0	8
	3	2	9

	p.	yds.	in.
38.	7	3½	11
	9	1	10
	4	2½	8
	6	1½	5

	po.	yds.	ft.	in.
39.	2	4	1	11
	12	3½	2	9
	9	4	1	7
	3	3½	1	9

	m.	fur.	po.	yds.
40.	14	2	15	2½
	22	5	37	5
	37	1	24	3
	43	6	21	1½

	m.	fur.	yds.	in.
41.	3	7	127	5
	6	7	77	6
	8	7	88	8
	5	2	165	1

	yds.	qrs.	n.
42.	14	3	2
	36	0	3
	54	1	1
	47	2	3

	yds.	qrs.	n.
43.	188	3	2
	297	0	1
	328	3	3
	169	1	2

	ells	qrs.	n.
44.	79	3	3
	67	4	1
	82	2	3
	89	3	2

	ells	qrs.	n.
45.	53	2	3
	24	0	3
	73	2	2
	24	4	3

	sq. yds.	sq. ft.	sq. in.
46.	20	8	100
	13	7	95
	14	6	43
	73	8	31
	15	8	49
	4	7	116

	r.	p.	sq. yds.
47.	8	33	20½
	7	14	13¼
	6	24	3½
	7	20	5¼
	14	3	18½
	15	4	7¼

	ac.	r.	p.
48.	37	2	14
	16	1	13
	9	2	17
	8	1	15
	17	3	29
	14	1	27

	p.	sq. yds.	sq. ft.	sq. in.
49.	3	14	6	47
	2	20¼	8	3
	5	13½	4	49
	6	2½	7	51
	8	29¼	4	17
	2	9	1	4

	a.	r.	p.	sq. yds.
50.	27	1	14	15½
	9	2	15	27¼
	11	3	27	4½
	15	1	10	11
	24	0	35	2½
	13	3	21	7

	r.	p.	sq. yds.	sq. in.
51.	28	23	13¼	131
	21	25	8	27
	18	7	20½	102
	4	3	28¼	46
	13	8	9½	72
	15	2	8¼	12

	c. yds.	c. ft.	c. in.
52.	131	25	278
	27	4	49
	5	18	151
	14	7	47
	23	15	129
	9	9	19

	c. yds.	c. ft.	c. in.
53.	128	26	1700
	4	18	215
	1	3	1507
	13	2	84
	117	16	101
	49	3	27

	galls.	qts.	pt.
54.	19	2	1
	31	1	0
	45	2	1
	73	3	1
	14	0	1
	15	2	1

	bush.	pks.	gall.
55.	14	3	1
	13	2	1
	25	1	1
	27	3	0
	14	2	1
	13	1	0

	qrs.	b.	pks.	gall.
56.	14	3	3	1
	32	4	1	0
	14	5	1	2
	52	3	3	0
	18	4	2	3
	17	3	1	3

	bush.	pk.	gall.	qt.
57.	13	1	1	1
	14	3	1	3
	71	2	0	2
	54	3	0	3
	17	2	1	1
	18	3	0	1

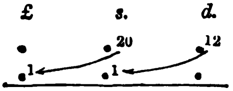
	d.	hrs.	min.	sec.
58.	4	14	39	14
	9	23	7	5
	18	5	18	18
	7	19	51	15
	12	3	8	6
	9	2	17	17

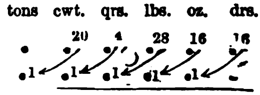
	yrs.	dys.	hrs.	min.
59.	4	150	23	17
	7	49	4	6
	18	5	20	20
	4	210	12	5
	13	77	7	7
	15	51	5	15

	yrs.	w.	d.	hrs.
60.	14	8	5	6
	7	18	6	15
	4	3	4	4
	15	2	6	23
	8	15	1	1
	17	7	7	7

COMPOUND SUBTRACTION.

50. Take *like from like*, and when necessary, add to the upper line an equivalent of 1 of the next higher denomination; then 1 of that next higher denomination must be added to the lower line, for the additions to both lines must be equal.





£	s.	d.	
14	7^{20}	8^{12}	$\frac{1}{4}^{4}$
5^1	18^1	10^1	$\frac{1}{2}$
8	8	9	3

tons	cwt.	qrs.	lbs.	oz.	drs.
3	1^{20}	1^{4}	2^{28}	5^{16}	8^{16}
2^1	3^1	2^1	18^1	6^1	9
	17	2	11	14	15

Ex. XXIX.

	£	s.	d.		£	s.	d.		£	s.	d.		£	s.	d.
1.	13	11	9	2.	54	13	8¾	3.	14	1	6½	4.	98	14	8
	4	6	5		37	9	7¼		13	8	5¼		47	5	9
5.	52	0	9	6.	31	13	0¼	7.	124	10	8½	8.	125	0	0
	47	10	11		17	15	6½		79	13	6¾		37	14	6¾
9.	225	13	0½	10.	235	11	0½	11.	317	15	0½	12.	478	0	0
	119	15	8¾		147	12	8¾		293	18	11¾		129	16	9½

lbs. oz. drs.	qrs. lbs. oz.	cwt. qrs. lbs.	qrs. lbs. oz.
13. 26 11 2	14. 12 0 2	15. 37 0 8	16. 1 15 1
13 7 5	4 12 14	12 2 17	1 14 8
lbs. oz. drs.	tons cwt. qrs.	cwt. qrs. lbs.	cwt. lbs. oz.
17. 19 2 13	18. 23 1 1	19. 17 1 3	20. 54 0 2
14 14 14	14 14 3	13 0 7	44 6 13
oz. dwt. grs.	oz. dwt. grs.	lbs. oz. dwt.	oz. dwt. grs.
21. 12 18 2	22. 23 7 21	23. 13 7 15	24. 11 0 0
8 14 6	18 9 22	6 11 18	2 18 22
oz. dwts. grs.	oz. dwts. grs.	oz. dwts. grs.	oz. dwts. grs.
25. 13 0 4	26. 27 0 0	27. 22 2 2	28. 24 0 2
1 15 20	0 11 13	13 12 12	17 13 21
drs. sc. grs.	oz. drs. sc.	lbs. oz. drs.	drs. sc. grs.
29. 6 1 17	30. 12 0 0	31. 38 7 1	32. 7 0 12
4 0 18	4 6 2	19 10 2	6 2 15
yds. ft. in.	p. yds. ft.	fur. p. yds.	m. fur. yds.
33. 14 0 6	34. 23 2 1	35. 3 27 1	36. 14 7 124
10 1 11	13 4½ 2	1 15 4½	8 7 129
m. fur. p.	fur. p. yds.	lea. m. fur.	fur. p. yds.
37. 14 0 6	38. 4 35 3	39. 27 0 0	40. 7 13 2½
11 6 19	3 36 4½	18 0 0	6 35 5
p. yds. ft.	yds. ft. in.	yds. qrs. nls.	ells qrs. nls.
41. 25 2 2	42. 33 0 0	43. 15 2 3	44. 15 1 1
15 3½ 1	16 2 9	13 3 2	11 2 3
sq. yds. sq. ft. sq. in.	p. sq. yds. sq. ft.	a. p. yds.	a. r. p.
45. 13 2 37	46. 23 14 5	47. 4 3 15	48. 27 1 28
6 8 120	13 20¼ 7	2 25 27¼	23 3 37
a. r. p.	r. p. sq. yds.	p. sq. yds. sq. ft.	sq. yds. sq. ft. sq. in.
49. 43 2 25	90. 3 23 21	51. 12 14¼ 5	52. 19 2 17
19 3 38	1 24 28¼	8 11 7	8 7 151
c. yds. c. ft. c. in.	c. yds. c. ft. c. in.	c. yds. c. ft. c. in.	c. yds. c. ft. c. in.
53. 18 15 225	54. 73 1 549	55. 45 14 249	56. 17 13 1
12 25 1310	7 23 1649	13 19 419	13 25 731
hrs. m. sec.	dys. hrs. m.	yrs. dys. hrs.	yrs. wks. dys.
57. 22 38 18	58. 14 17 19	59. 31 121 15	60. 15 35 3
9 41 35	6 22 28	19 300 18	18 47 5

COMPOUND MULTIPLICATION.

51. Multiply each part of the multiplicand, beginning with the lowest denomination and changing (Reduction) into the next higher denomination, as opportunity occurs, leave behind like with like, and carry on like to like.

£	s.	d.		
14	18	10	10*d.* × 5 = 50*d.* = 4*s.* 2*d.*	38.
		5	18*s.* × 5 = 90*s.* + 4*s.* = 94*s.* = £4 14*s.*	38.
			£14 × 5 = £70 + £4 = £74.	
74	14	2		

Ex. XXX.

	£	s.	d.			£	s.	d.	
1.	5	8	3	× 2	2.	27	13	6	× 3
3.	39	13	7	× 3	4.	25	16	10	× 2
5.	44	1	7	× 4	6.	73	16	9¼	× 4
7.	99	16	8	× 5	8.	57	13	2½	× 5
9.	213	13	6	× 6	10.	145	11	4¼	× 6
11.	143	6	7	× 7	12.	161	13	8½	× 7
13.	105	14	8	× 8	14.	137	18	5¼	× 8
15.	115	17	6	× 9	16.	187	19	6½	× 9
17.	214	3	10	× 10	18.	157	5	10¾	× 10
19.	115	12	4	× 11	20.	15	17	11½	× 11

52. When the multiplier can be broken up into two or three factors, each being not more than 12, the factors may be used successively as multipliers.

£	s.	d.			
14	1	8¼	× 135 = 5 × 3 × 9		29.
		5			
70	8	5¼	= 5 times.		
		3			
211	5	3¾	= 15 times.	(20).	
		9			
1901	7	9¾	= 135 times.	(20).	

Ex. XXXI.

	£	s.	d.			£	s.	d.	
1.	13	15	6½	× 16	2.	75	15	7½	× 18
3.	18	18	7¾	× 21	4.	39	16	10½	× 35

	£	s.	d.			£	s.	d.	
5.	79	9	$9\frac{1}{2}$	× 28	6.	79	18	$7\frac{3}{4}$	× 42
7.	23	3	$8\frac{3}{4}$	× 56	8.	29	11	$6\frac{1}{2}$	× 88
9.	15	15	$7\frac{1}{4}$	× 121	10.	87	17	$0\frac{3}{4}$	× 132
11.	44	4	$6\frac{3}{4}$	× 144	12.	15	19	$9\frac{1}{2}$	× 120

53. With fractional parts other than halves (half-pence) and fourths (farthings) the method of working is the same.

54. Table of fractional parts: 2 halves = 1 integer; 3 thirds = 1 int.; 4 fourths = 1 int.; 5 fifths = 1 int.; 6 sixths = 1 int.; 7 sevenths = 1 int.; 8 eighths = 1 int., &c. (25).

£ *s.* *d.* — £ *s.* *d.* *thirds.*

2 3 $10\frac{2}{3}$ × 7 otherwise written 2 3 10 2 × 7

(25) 3 thirds = 1 int.
(38) *No.* of int. pence *one-third of No.* of thirds of a penny.

```
3)14 thirds          |   £  s.  d. thirds.
   4 int. 2 thirds.  |   2  3  10    2
                     |               7
                     |   ----------------
                     |  15  7   2    2 i.e. £15 7s. 2⅔d.
```

Ex. XXXII.

	£	s.	d.			£	s.	d.	
1.	5	8	$6\frac{1}{7}$	× 8	2.	3	3	$9\frac{2}{5}$	× 9
3.	8	4	$8\frac{1}{6}$	× 11	4.	2	5	$8\frac{3}{8}$	× 7
5.	7	17	$11\frac{1}{3}$	× 8	6.	13	15	$10\frac{2}{7}$	× 9
7.	13	2	$9\frac{3}{5}$	× 11	8.	8	2	$10\frac{1}{11}$	× 12

55. When the multiplier is a large number, or one not easily resolved (by division) into factors (29), it is usual to multiply as in the first case.

```
 £  s.  d.
14  18  6¾ × 367          3 f. × 367 = 1101 far. = 275¼d.
        367
-------------             6d. × 367 = 2202d.
5478 12  5¼                            275d. added.
                                       -----
                                       2477d. = 206s. 5d.
```

£	s.	d.	
14	18	6¾	× 367
		367	
5478	12	5¼	

18s. i.e. (10 + 8) × 367 = 3670 }
2936 }
206 added.
6812s. = £340 12s.

£14 i.e. (10 + 4) × 367 = 3670 }
1468 }
340 added.
£5478

56. Otherwise the multiplication might have been worked out according to the following method—

	Times	Times	Times
367 =	300 =	100 × 3 =	10 × 10 × 3
	60 =	10 × 6	
	7		
	367		

£ s. d.
14 18 6¾ × 7
10
10 times × 6
10
100 times
3
300 times
60 „
7 „
367 times

Ex. XXXIII.

£ s. d.
1. 4 13 8½ × 29
2. 4 14 6¾ × 37
3. 32 17 9¼ × 46
4. 55 18 9½ × 76
5. 18 12 6¾ × 107
6. 23 13 7¾ × 193
7. 4 15 11½ × 327
8. 18 18 8½ × 276
9. 55 4 8¾ × 723
10. 8 17 9¾ × 85
11. 2 qrs. 7 lbs. 15 oz. 15 drs. × 8
12. 5 tons 14 cwt. 18 lb. 4 oz. × 9
13. 3 tons 23 cwt. 27 lbs. 4 drs. × 21
14. 8 tons 15 cwt. 1 qr. 7 lbs. × 36
15. 8 cwt. 2 qrs. 2 lbs. 2 oz. 2 drs. × 49
16. 7 tons 1 qr. 2 lbs. 13 drs. × 54
17. 2 lbs. 5 oz. 14 dwts. 12 grs. × 55
18. 3 lbs. 7 oz. 8 dwts. 21 grs. × 63
19. 4 fur. 27 yds. 1 ft. 8 in. × 56

20. 7 fur. 15 yds. 1 ft. 2 in. × 87.
21. 4 ac. 3 r. 19 p. × 72.
22. 15 ac. 1 r. 13 p. × 84.
23. 2 sq. yds. 9 sq. ft. 15 sq. in. × 110.
24. 8 cub. yds. 13 cub. ft. 27 cub. in. × 50.
25. 78 galls. 2 qts. 1 pt. × 108.
26. 17 qrs. 105 d. 12 h. 10 sec. × 144.

COMPOUND DIVISION.

57. Divide the highest denomination; reduce any remainder that may occur to the next lower denomination, add any of the same name and divide again, following out the same method until the lowest denomination has been reached. If the divisor can be broken up into two or more factors (29) each being less than 12, the factors may be used as separate and successive divisors.

$$\begin{array}{r|rrr} & £ & s. & d. \\ 2) & 41 & 5 & 10 \\ \hline & 20 & 12 & 11 \end{array}$$

rem. £1 = 20*s.* + 5*s.* = 25*s.*
„ 1*s.* = 12*d.* + 10*d.* = 22*d.*

$$\begin{array}{r|rrr} & £ & s. & d. \\ 3) & 17 & 18 & 8\frac{1}{4} \\ \hline & 5 & 19 & 6\frac{3}{4} \end{array}$$

rem. £2 = 40*s.* + 18*s.* = 58*s.*
„ 1*s.* = 12*d.* + 8*d.* = 20*d.*
2*d.* = 8 f. + 1 f. = 9 f.

$$\begin{array}{rllllll} & £ & s. & d. & £ & s. & d. \\ 17) & 896 & 4 & 8\frac{1}{8} & (52 & 14 & 4\frac{97}{136} \\ & 85 \\ & \text{—} \\ & 46 \\ & 34 \\ & \text{—} \\ & 12 \\ & 20 \\ \hline 17) & 244\ (14 \\ & 17 \\ & \text{—} \\ & 74 \\ & 68 \\ & \text{—} \\ & 6 \\ & 12 \\ \hline 17) & 80\frac{1}{8}\ (4 \\ & 68 \\ & \text{—} \\ & 12\frac{1}{8}d.\ \text{rem}^{\text{s}}. \\ & \text{undivided.} \end{array}$$

Since 1 integer = 8 eighths (25); 12 integers = 96 eighths; and $12\frac{1}{8}$ = 97 eighths; a change of form to work together the whole pence, 12, and the fraction of another penny, $\frac{1}{8}$.

Now 97 *eighths* when *divided by* 8 (to get them back to integers) *and also by the divisor* 17, will not give even 1 as a result; the DIVISION is therefore ONLY EXPRESSED.

$$\begin{array}{r} 8)\ 97 \\ \hline 17)\ \underline{} \end{array} = 136)\ 97 \quad \text{EXPRESSED } \frac{97}{136}. \quad (81b).$$

Divide £13 8*s.* 10$\frac{1}{7}$*d.* by 45 5×9.

$$\begin{array}{rrrl} & £ & s. & d. \\ 5) & 13 & 8 & 10\frac{1}{7} \\ \hline & 2 & 13 & 9 \ldots\ldots 1\frac{1}{7}d. \text{ remain}^{s} \text{ undivided} \end{array}$$

1*d.* = 7 sevenths (25).
1$\frac{1}{7}$*d* = 8 „

Now divid^{s} 8 sevenths by 7 (to get back to integers) and also by the given divisor 5, we have

$$\begin{array}{l} 7\underline{)8} \\ 5\underline{)\quad} \end{array} \quad \text{i.e. } 35\underline{)\ 8} \quad \text{WRITTEN } \frac{8}{35}. \quad (81b).$$

$$\begin{array}{rrrl} 9) & 2 & 13 & 9\frac{8}{35} \\ \hline & & 5 & 11 \ldots\ldots 6\frac{8}{35}d. \text{ remain}^{s} \text{ undivided} \end{array}$$

1*d.* = 35 thirty-fifths (25).
6*d.* = 210 „
6$\frac{8}{35}$*d.* = 218 „

Now divid^{s} 218 thirty-fifths by 35 (to get back to integers) and also by the given divisor 9, we have

$$\begin{array}{l} 35\underline{)\ 218} \\ 9\underline{)\qquad} \end{array} \quad \text{i.e. } 315\underline{)\ 218} \quad \text{WRITTEN } \frac{218}{315}. \quad (81b).$$

$$\begin{array}{rrrr} & £ & s. & d. \\ 5) & 13 & 8 & 10\frac{1}{7} \\ \hline 9) & 2 & 13 & 9\frac{8}{35} \\ \hline & & 5 & 11\frac{218}{315} \end{array}$$

Ex. XXXIV.

	£	s.	d.		
1.	14	14	3½	÷	2.
2.	15	14	3	÷	3.
3.	72	15	8	÷	4.
4.	71	7	2¼	÷	5.
5.	79	10	3	÷	6.
6.	30	12	4¼	÷	7.
7.	38	14	0	÷	8.
8.	97	18	10½	÷	9.
9.	13	16	0¾	÷	10.
10.	47	18	10	÷	11.
11.	105	10	6	÷	12.
12.	276	19	9	÷	12.

Ex. XXXV.

1. £17 18*s.* 10*d.* ÷ 23.
2. £492 15*s.* 8*d.* ÷ 146.
3. £19 14*s.* $6\frac{1}{8}$*d.* ÷ 17.
4. £115 14*s.* $11\frac{3}{8}$*d.* ÷ 19.
5. £275 18*s.* $11\frac{3}{7}$*d.* ÷ 29.
6. £357 18*s.* $11\frac{5}{7}$*d.* ÷ 59.
7. 81 c. 1 qr. 16 lbs. 5 oz. ÷ 174.
8. 98 c. 7 lbs. 8 oz. $8\frac{5}{8}$ drs. ÷ 371.
9. 15 t. 2 c. 2 qr. 8 lbs. $5\frac{1}{3}$ oz. ÷ 74.
10. 1 t. 1 lb. 1 oz. $1\frac{1}{10}$ drs. ÷ 757.
11. 513 m. 1 fur. 9 p. ÷ 119.
12. 87 m. 3 f. 8 p. ÷ 137.
13. 52 w. 1 d. 14 hrs. $9\frac{1}{3}$ m. ÷ 82.
14. 1 y. 355 d. 5 hrs. 48 m. 48 sec. ÷ 731.
15. 300 a. 1 r. 18 p. ÷ 193.
16. 9 m. 2 fur. 20 p. 9 yds. 1 ft. ÷ 877.
17. £218 18*s.* $11\frac{3}{14}$*d.* ÷ 999.
18. £78 18*s.* $9\frac{1}{9}$*d.* ÷ 457.

58. Should the divisor be 10, 100, 1000, &c., it becomes a question of the number of tens, hundreds, thousands, &c., and the number can be read off at sight, remembering that in any number of *hundreds*, the *last figure is* always *hundreds*, of thousands, thousands, &c. (8).

$$\begin{array}{rrlll}
 & \text{h} & s. & d. & \\
 & £371{,}06 & 4 & 9 \div 100 & \\
 & 20 & & & \\
 & \overline{1{,}24} & & & \\
 & 12 & & & \\
£371\quad 1\quad 2\frac{97}{100} & \overline{2{,}97} & \text{not divided, but division} & & \\
 & & \text{expressed } \frac{97}{100}\ (57) & &
\end{array}
\qquad
\begin{array}{rll}
\text{th} & s.\ d. & \\
£36{,}528 & 6\ 11 \div 1000 & \\
20 & & \\
\overline{10{,}566} & & \\
12 & & \\
 & 36\ 10\ 6\frac{803}{1000} & \\
\overline{6{,}803} & &
\end{array}$$

Ex. XXXVI.

1. 353 13 4 ÷ 10. 2. 66 12 6 ÷ 10.
3. 987 3 9 ÷ 100. 4. 294 11 8 ÷ 100.
5. 7557 5 10 ÷ 1000. 6. 291 13 4 ÷ 1000.
7. 16458 6 8 ÷ 10000. 8. 2125 0 0 ÷ 100.

59. To find how many times one sum of money occurs in another, or one weight in another, &c., reduce both to the same denomination, and substituting abstract for the concrete numbers, the result will be found by division.

s. *d.* £ *s.* *d.*
How often do 6 8 occur in 12 13 4
12 20
— —
80*d.* 253 $8\cancel{0})\underline{304\cancel{0}}$ dividing first by 10
12 38 and then by 8.
——
3040*d.*

Ex. XXXVII.—How often is contained

1. £2 10*s.* 7½*d.* in £34 3*s.* 6*d.*?
2. £11 7*s.* 9¾*d.* in £136 13*s.* 9*d.*?
3. £22 15*s.* 7½*d.* in £205 0*s.* 7½*d.*?
4. 6 c. 2 qrs. 12 lbs. in 178 c. 1 qr. 16 lbs.?
5. 17 c. 1 qr. 17 lbs. in 155 c. 2 qrs. 13 lbs.?
6. 1 yd. 1 ft. 6 in. in 100 miles?
7. 1 c. 1 qr. 1½ lbs. in 1 ton?
8. 15 m. 1 f. 18 p. in 1624 m. 3 f. 6 p.?
9. 1 qr. 18 lbs. in 100 cwt.?
10. £12 17*s.* 6*d.* in £100?

MISCELLANEOUS QUESTIONS (XVII–XXXVII).

1. Reduce £413 to halfpence. 38.
2. In 21746385 f. how many £? 38.
3. Reduce 2 tons 1 c. 1 qr. 5 lb. to ounces. 39.
4. In 12712560 drs. how many cwt.? 39.
5. In 21745628 lbs. how many tons, cwt., &c.? 39.
6. In 2 lbs. 1 sc. 14 grs. how many grs. Ap.? 42.
7. Reduce 2 m. 1 f. 4 p. 2 yds. 0 ft. 6 in. to inches. 43.
8. Reduce 3 m. 3 f. 1 p. 3 yds. 2 ft. 11 in. to inches. 43.
9. Reduce 5217632518 inches to miles. 43.
10. Reduce 34716284 inches to miles. 43.
11. In 413 guineas 15*s.* 6*d.* how many sixpences? 38.
12. In 51437298*d.* how many guineas? 38.
13. Reduce 4 a. 1 r. 2 p. 15. sq. yds. 1 sq. ft. 17 sq. in. to inches. 44.
14. In 5431256 sq. yds. how many perches? 44.
15. Reduce 1 a. 3 r. 28 p. 18 sq. yds. 8 sq. ft. 39 sq. in. to inches. 44.
16. In 31254721 sq. in. how many acres? 44.
17. Reduce 141523876 cub. in. to yards. 45.
18. In 17 c. yds. 1 c. ft. 35 c. in. how many inches? 45.

19. In 4716 guineas how many £? 48.
20. In £37215 17*s.* 6*d.* how many half-crowns? 48.
21. In 21543786 sec. how many years? 47.
22. In 9 d. 15 hrs. 8 min. 12 sec. how many sec.? 47.
23. In 218 galls. how many pints? 46.
24. In 5 qrs. 1 b. 1 pk. how many galls.? 46.
25. Reduce 12345678 galls. to qts.? 46.
26. Reduce 5 oz. 13 dwt. 16 grs. Troy to grains. 41.
27. In 15 c. 3 qrs. 19 lbs. how many lbs.? 40.
28. In 4715268 oz. how many cwt. &c.? 39.
29. In 21435176 inches how many yards? 43.
30. In 378 guineas how many £? 48.
31. Multiply £4 17*s.* 8*d.* by 35. 29, 52.
32. Multiply £2 8*s.* 11$\frac{1}{2}$*d.* by 47. 55 or 56.
33. Multiply 1 c. 1 qr 4 lbs. 7 oz. 2 drs. by 9. 51.
34. Multiply 2 c. 3 qrs. 6 lbs. 3 oz. by 59. 55 or 56.
35. Multiply £3 8*s.* 4$\frac{1}{4}$*d.* by 478. 56.
36. Divide £18 1*s.* 6*d.* by 8. 57.
37. Divide £38 15*s.* 4*d.* by 18 (factors). 29, 57.
38. Divide £139 16*s.* 10$\frac{1}{2}$*d.* by 189. 57.
39. Divide £377 2*s.* 11$\frac{3}{7}$*d.* by 78. 57.
40. Divide £4 2*s.* 10$\frac{1}{6}$*d.* by 5. 54.
41. Multiply £3 16*s.* 2$\frac{1}{2}$*d.* by 315 (factors). 29, 55.
42. Multiply £2 17*s.* 4$\frac{3}{7}$*d.* by 9. 54.
43. Multiply £1 15*s.* 4$\frac{5}{9}$*d.* by 12. 54.
44. Multiply 1 c. 1 qr. 4$\frac{2}{3}$ lbs. by 21 (factors). 29, 54.
45. Multiply 3 qrs. 5 lbs. 6$\frac{3}{8}$ oz. by 28 (factors). 29, 54.
46. Divide £7 1*s.* 11$\frac{2}{3}$*d.* by 28. 57.
47. Divide £15 13*s.* 9$\frac{5}{11}$*d.* by 144. 57.
48. Divide £3 17*s.* 10$\frac{7}{10}$*d.* by 16. 57.
49. Divide £214 4*s.* 8$\frac{1}{12}$*d.* by 121. 57.
50. Divide £39 1*s.* 6$\frac{1}{6}$*d.* by 7. 57.

MEASUREMENT OF RIGHT-ANGLED SURFACES.

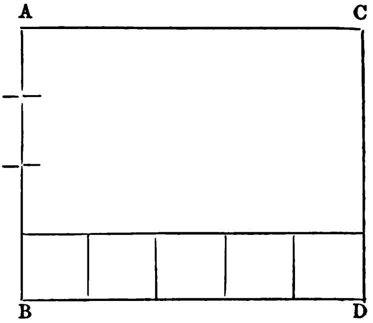
Definitions.

60. I. A line is length without breadth. In measuring a line, the standards of reference are *lines* of fixed length, named, inch, foot, yard, pole, furlong, mile.

61. II. A surface, or superficies, has only length and breadth. The boundaries of a superficies are lines.

62. In measuring a surface, the standards of reference are *square portions of surface* of definite size, as square inch, square foot, square yard, &c.

63. III. A square is a four-sided figure, which has all its sides equal, and all its angles right angles.



64. Let the rectangular figure A B C D represent a surface, 5 inches in length and 4 inches in breadth.

If a square be described on each of the five line inches of the length boundary B D, we have 5 square inches in a row, and it will be seen by reference to the figure that we can have just as many rows as there are line inches in the width boundary A B; hence, when the length and breadth boundaries are marked off alike, i.e., both in line inches, line feet, &c., we can get the whole number of square inches, square feet, &c., of surface in the following manner—

(*a*) No. in the length tells No. of squares in 1 row, } and
(*b*) „ „ „ breadth „ „ „ rows, } *vice versâ.*

In this example
a length of 5 inches gives 5 square inches in 1 row,
a breadth of 4 inches „ 4 rows,
i.e. 4 times 5 sq. in. = 20 sq. in.

65. Every calculation of the measurement of a rectangular surface must be put into the form:
"*so many squares in* 1 *row;*" "*so many rows,*" and *worked* out *by multiplication.*

$$(a)\ \begin{array}{c}\text{No. of squares}\\ \text{in 1 row.}\end{array} \times \begin{array}{c}\text{No. of}\\ \text{rows.}\end{array} = \begin{array}{c}\text{No. of squares}\\ \text{altogether,}\end{array}$$

and as any two of the above being given, the third can be found (29), then

$$(b)\ \begin{array}{c}\text{No. of squares}\\ \text{altogether}\end{array} \div \begin{array}{c}\text{No. of}\\ \text{rows}\end{array} = \begin{array}{c}\text{No of squares}\\ \text{in 1 row,}\end{array}\ \text{which tells the } \textit{length.}$$

$$(c)\ \begin{array}{c}\text{No. of squares}\\ \text{altogether}\end{array} \div \begin{array}{c}\text{No. of squares}\\ \text{in 1 row}\end{array} = \begin{array}{c}\text{No. of}\\ \text{rows,}\end{array}\ \text{which tells the } \textit{breadth.}$$

$(a)\ 5 \times 4 = 20.$ $(b)\ 20 \div 4 = 5.$ $(c)\ 20 \div 5 = 4.$

Ex. XXXVIII. (*a*) Find the surface measure from the following line measurements.

1. Length 15 ft., breadth 12 ft. (i.e. 15 sq. ft. in 1 row and 12 rows).
2. „ 13 ft. 6 in., breadth 10 ft. 2 in. (162 sq. in. in 1 row and 122 rows).
3. Length 13 yds., breadth 12 yds.
4. „ 4 yds. 2 ft., breadth 4 yds. 2 ft.

Ex. XXXIX. (*b*) Find the length from the following:

1. Surface 11 sq. yds. 3 sq. ft. 31 sq. in., width 2 ft. 9 in.
2. „ 80 sq. yds. 6 sq. ft. 88 sq. in. „ 3 ft. 5 in.
3. „ 1 sq. yd. 6 sq. ft. 50 sq. in. „ 1 yd. 1 in.
4. „ 22 sq. yds. 3 sq. ft. 64 sq. in. „ 3 yds.

Ex. XL. (*c*) Find the width from the following:

1. Surface 11 sq. yds. 3 sq. ft. 30 sq. in., length 37 ft. 2 in.
2. „ 80 sq. yds. 6 sq. ft. 64 sq. in. „ 23 ft.
3. „ 1 sq. yd. 6 sq. ft. 60 sq. in. „ 1 yd. 2 ft.
4. „ 22 sq. yds. 3 sq. ft. 108 sq. in. „ 3 yds.

Ex. XLI.

1. What is the square measure of a tennis court 98 ft. long and 37 ft. wide? 65*a*.

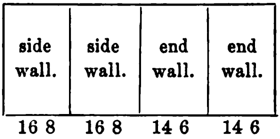
2. What is the length of a surface which is 2 ft. 9 in. wide and which measures 5 sq. yds. 0 sq. ft. 21 sq. in.? 65*b*.

3. If the measurements of the surface of a wall is 289 sq. ft. 108 sq. in., and its height (side boundary) is 9 ft. 6 in., what is its length? 65*b*.

4. If the four walls of a room have 578 sq. ft. 96 sq. in. of surface, and their united lengths make up 56 ft., what is their height (side boundary)? 65 (*c*).

(*x*) How many square yards of wall will be papered in a room 16 ft. 8 in. long, 14 ft. 6 in. wide, and 10 ft. 4 in. high?

Supposing the four walls to be placed side by side.



height
10 ft. 4 in. = 124 in.

= 62 ft. 4 in. = 748 in.

748 in. and 124 in. give (64 *a b*) 748 sq. in. in 1 row, and 124 rows.

124 times.

2992
1496
748

(29*) 144 | 12) 92752 sq. in. altogether.
12) 7729 – 4
9) 644 – 1 | 16 sq. in. (30 *c*).
71 sq. yds. 5 sq. ft. 16 sq. in.

5. How many sq. ft. of wall will be painted, when the length, breadth, and height of a room are 17 ft. 6 in., 12 ft. 8 in., and 9 ft. 6 in. respectively? XLI. (*x*).

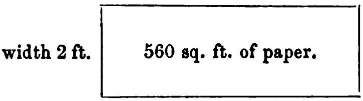
6. How much wainscoting will there be in a square room, the side of which measures 16 ft. 6 in., the height being 10 ft. 8 in.? XLI. (*x*).

7. The measurements of a room are, as follows, 18 ft. 6 in. long, 12 ft. 6 in. wide, and 10 ft. 8 in. high; the walls are to be in part covered with wainscoting 2 ft. 6 in. high, how many sq. ft. will be covered? XLI. (*x*).

(*y*) What length of paper, 2 ft. wide, will be required for a room 16 ft. long, 12 ft. wide, and 10 ft. high?

16 + 16 + 12 + 12 = 56 ft. = length of 4 walls taken together.
10 ft. = height.

i.e. (64 *a*, *b*) 56 sq. ft. in 1 row, and 10 rows = 560 sq. ft. of wall and *also* 560 *sq. ft. of paper*, for the surfaces of walls and paper are alike.



i.e. 560 sq. ft. in 2 rows (65 *b*) = 280 sq. ft. in 1 row, and from this we know (64 *a*, *v v.*) that *the line* on which the 280 sq. ft. are described *measures* 280 *lineal feet* = 93 yds. 1 ft.

8. What length of carpet, 2 ft. 6 in. broad, will cover a room 36 ft. by 28 ft. 65*a*, 65*b*.

9. Find the length of a passage, 4 ft. 6 in. wide, which is covered by 8 sq. yds. 2 sq. ft. 36 sq. in. of oil-cloth. 65*a*, 65*b*.

10. How many yards of matting, 3 ft. 3 in. wide, will be required for a square room, whose side measures 18 ft. 6 in.? 65*a*, 65*b*.

11. How many yards of carpet, 16 in. wide, will be required for a floor 18 ft. 6 in. long and 14 ft. 8 in. broad? 65*a*, 65*b*.

12. A room measuring 18 ft. 6 in. long, 12 ft. 6 in. broad, and 10 ft. 8 in. high, is to be papered; how many yards will be required if the paper is 2 ft. wide? Ex. XLI. (*y*). 65*a*, 65*b*.

13. The measurements of a room are, length 16 ft. 3 in., breadth 14 ft. 9 in., and height 10 ft. 6 in.; how many yds. of paper 2 ft. 4 in. wide will be required for it? Ex. XLI. (*y*). 65*a*, 65*b*.

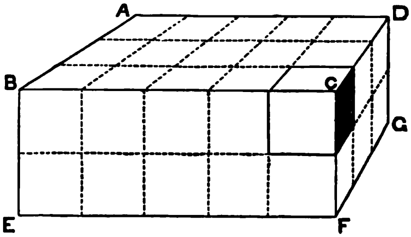
14. The length and breadth of a room are 18 ft. 6 in. and 16 ft. 8 in., how many yards of carpet 1 yd. 1 ft. 2 in. wide will be required to cover it? 65*a*, 65*b*.

MEASUREMENT OF RECTANGULAR SOLIDS

66. A cube is a solid body, having equal length, breadth, and thickness. It has six surfaces (61), which are squares (63).

67. In measuring a solid body, the standards of reference are solid bodies of a fixed size, as cubic inch, cubic foot, cubic yard, &c.

68. Let the accompanying figure represent a block of stone, 5 ft. in length (B C), 3 ft. in width (A B), and 2 ft. in thickness (B E).



Of the surface A B C D, 5 sq. ft. in a row (64*a*), and 3 rows (64*b*), give 15 sq. ft. (65*a*); now *each of* these *square feet* will become the *top of a cube*, if we take a thickness of 1 foot, and so doing we have *a set of* 15 *cubic feet;* in the same way the next foot of thickness will give us a second set of 15 cubic feet, and so on, hence

(*a*) No. in length	tells	No. of squares in a row. 64*a*.
(*b*) No. in breadth	„	No. of rows. 64*b*.
(*c*) *No. of squares of top surface* (65*a*) *tells No. of cubes in* 1 *set.*		
(*d*) No. in thickness	tells	No. of sets of cubes.

No of squares in 1 row.		No of rows.						No. of sets of cubes.		Whole No. of cubes.
5	×	3	=	No. of squares of top surface.	=	No. of cubes in 1 set.	×	2	=	30

69. And of such four, $5 \times 3 \times 2 = 30$; *i.e.* no. in length × no. in breadth × no. in thickness = no. of cubes, any three being given, the fourth can be found by division (29).

a) $? \times 3 \times 2 = 30$. (*b*) $5 \times ? \times 2 = 30$. (*c*) $5 \times 3 \times ? = 30$.

Find the cubic content of a piece of stone 20 ft. long, 10 ft. 4 in. broad, and 3 ft. 8 in. thick.

Length = 240 in.	gives	240 sq. inches in a row.	68*a*.
Breadth = 124 in.	„	124 rows	68*b*.
Thickness = 44 in.	„	44 sets of cubic inches.	68*d*.

240 sq. in. 68*a*.
124 times. 68*b*.

960
480
240

29760 sq. in. of top
✱ surface. 68*c*.

✱ 29760 cub. in. in 1 set of cubes. 68*c*.
44 times. 68*d*.

119040
119040

1728) 1309440 cub. in. altogether.
27) 757 cub. ft. 1344 cub. in.
28 cub. yds. 1 cub. ft. 1344 cub. in.

Ex. XLII.

Find the cubic content with the following measurements.

1. Length 17 ft. 6 in., breadth 8 ft. 5 in., thickness 3 ft. 9 in. 68.
2. „ 12 ft. 8 in., „ 4 ft. 6 in., „ 2 ft. 5 in. 68.
3. „ 8 ft. 10 in., „ 2 ft. 9 in., „ 4 ft. 7 in. 68.

With the following measurements find (4) length, (5) width, and (6) thickness.

4. Cubic content 15 cub. yds. 2 ft. 1673 in., width 7 ft. 5 in., thickness 7 ft. 5 in. 69*a*.

5. Cubic content 13 cub. yds. 15 ft. 1152 in., length 9 ft. 2 in., thickness 3 ft. 4 in. 69*b*.

6. Cubic content 14 cub. yds. 12 ft. 1080 in., length 11 ft. 3 in., width 10 ft. 5 in. 69*c*.

7. How many cubic feet of air are there in a room 12 ft. 6 in. long, 10 ft. 4 in. broad, 10 ft. 2 in. high? 68.

8. How many cubic feet of timber in a piece 13 ft. long, 8 ft. wide, and 3 ft. 6 in. thick? 68.

9. A beam contains 725 cub. ft., its width and thickness are each 2 ft. 6 in., find its length? 69*a*.

10. A tank measuring 18 ft. long, 8 ft. wide, and 10 ft. deep, is filled with water to a depth of 3 ft.; how many cubic feet of water in the tank? 68.

11. What must be the width of a trench 120 ft. long and 5½ ft. deep, that it may contain 1650 cubic ft.? 69*b*.

12. What is the length of a room which contains 30240 cub. ft. of air, the width being 10 ft. and the height 9 ft.? 69*a*.

13. How many square yards of carpet will be required for a room 24 ft. 8 in. long 15 ft. 6 in. broad? find also its cost at 3*s*. 6*d*. per. sq. yd. 64, 65*a*.

14. A room is 30 ft. by 22 ft., what will be the expense of covering it with carpet ¾ yd. broad, at 3*s*. 9*d*. per yard. 65*a*, 65*b*.

GREATEST COMMON MEASURE.

70. The Greatest Common Measure of two or more numbers is the greatest number which will *divide* them without a remainder.

71. TO FIND THE GREATEST COMMON MEASURE OF TWO NUMBERS divide the greater by the less; then, making the remainder a divisor, and the last divisor a dividend, divide again and continue according to that plan, until there is no remainder. The *last divisor* will be the *Greatest Common Measure.*

Find the G. C. M. of 558 and 12555.

```
558) 12555 (22
     1116
     ----
      1395
      1116
     ----
      279) 558 (2        G. C. M. = 279.
           558
           ---
```

Ex. XLIII. Find the G. C. M. of

1. 336 and 448.
2. 522 and 2408.
3. 75 „ 232.
4. 1250 „ 525.
5. 1515 „ 2121.
6. 381 „ 1143.
7. 516 „ 1806.
8. 450 „ 6948.
9. 2135 „ 2275.
10. 525 „ 10000.
11. 3834 „ 10224.
12. 3605 „ 4635.

72. *To find the G. C. M. of three or more numbers*, find the G. C. M. of any two of the given numbers; taking the G. C. M. obtained, and another of the given numbers, find their G. C. M., and so work on, using the last G. C. M. obtained, and some one of the given numbers until all have been used. The *last G. C. M. obtained* will be the *the Greatest Common Measure of all* the numbers.

Find the G. C. M. of 968, 5324, and 3388.

```
      968) 5324 (5            G. C. M. 484) 3388 (7
           4840                             3388
G. C. M. 484) 968 (2                        ----
              968
              ---
```

Ex. XLIV. Find the G. C. M. of

1. 28, 144 and 1728.
2. 336, 448 and 560.
3. 133, 171 „ 152.
4. 120, 175 „ 210.
5. 1211, 1384 „ 1557.
6. 127, 139 „ 75.

LEAST COMMON MULTIPLE.

73. One number is a multiple of another when it contains that number as one of its factors.

74. The Least Common Multiple of two or more numbers is the least number in which they all occur as factors.

75. TO FIND THE LEAST COMMON MULTIPLE OF TWO OR MORE NUMBERS, set the numbers in a line, and continue to take out by Division any factors common to two or more of them, until there remain only the factors which still occur once. The Least Common Multiple will be found to be the number made up of *all* the *factors* that occur, *taken once only*.

Find the L. C. M. of 15, 16, 40.

$$\begin{array}{rlll}
\text{once } 5) & \overset{5\times3}{15} & 16 & \overset{5\times8}{40} \\ \hline
\text{once } 8) & 3 & \overset{8\times2}{16} & \overset{8\times1}{8} \\ \hline
 & 3 & 2 & 1
\end{array}$$

$$5 \times 8 \times 3 \times 2 \times 1 = 240$$

The factor 5 occurs twice in the given numbers, but it must only enter once into the Least C. M., it is therefore placed on the outside to have it once, and taken away wherever it occurs; the same course is followed to have the factor 8 once only, and then there still remain 3, 2, 1, which do not contain any common factor, except 1, hence the factors, 5, 8, 3, 2, 1 give the L. C. M. i.e. 240.

76. One given number contained in another may be struck out at first, for no factor can occur more than once in the Least Common Multiple.

Find the L. C. M. of

$$\begin{array}{rllllll}
2) & \cancel{2} & \cancel{4} & \cancel{6} & 10 & 12 & 14 \\ \hline
 & & & & 5 & 6 & 7
\end{array}$$

$$2 \times 5 \times 6 \times 7 = 420$$

L. C. M.

Ex. XLV. Find the L. C. M. of

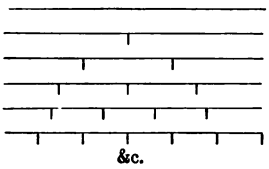
1. 20, 25.
2. 21, 28.
3. 4, 6, 8.
4. 7, 14, 33.
5. 12, 14, 15, 16.
6. 8, 10, 12, 14.
7. 3, 6, 8, 9, 12.
8. 5, 4, 6, 8, 9.
9. 2, 3, 4, 5, 6, 7.
10. 24, 42, 56.
11. 6, 10, 16, 18, 21.
12. 9, 8, 7, 6, 5, 4, 3, 2.
13. 2, 3, 9, 10, 11, 12.
14. 12, 14, 15, 16, 18, 20.

VULGAR FRACTIONS, OR FRACTIONS IN COMMON USE.

77. When an integer or whole thing is divided into a number of *equal* parts, those parts are called Fractions.

78. The *names* given to the equal parts *are taken from the number of parts*, into which the integer is divided.

Taking a line as a representative integer.



	Name of parts.	Written.
Undivided		$\frac{1}{1}$
2 equal parts	halves	$\frac{1}{2}$
3 ,, ,,	thirds	$\frac{1}{3}$
4 ,, ,,	fourths	$\frac{1}{4}$
5 ,, ,,	fifths	$\frac{1}{5}$
6 ,, ,,	sixths	$\frac{1}{6}$

79. It is to be observed that the Integer is equal to 2 halves, 3 thirds, 4 fourths, 5 fifths, 6 sixths, &c., and this Table of Fractional parts is necessary for Reduction of integers to parts, and of parts to integers, and must be used as other Tables. 38.

80. A Fraction is written thus:

$$\frac{\text{number of equal parts of an integer.}}{\text{name of equal parts, \textit{expressed by figures.}}}$$

In $\frac{3}{5}$, the lower figure 5, called the Denominator, in showing that the integer is divided into 5 equal parts, gives the *name fifths;* the upper figure 3, called the Numerator, gives the *number* of fifths.

81. Since $\frac{3}{4}$ as $\frac{\text{number}}{\text{name}}$ expresses 3 fourths of an integer, and also expresses the result of 3 integers divided by 4.

1 int. = 4 fourths (78). 3 int^{s}. = 12 ,,	$4\underline{)12}$ fourths or 3 integers. 3 fourths.

it is to be noted that a fraction may be regarded as

(*a*) $\frac{\text{number}}{\text{name}}$ of eq. pts. of int. (*b*) $\frac{\text{dividend}}{\text{divisor}}$ (*c*) a form by which division may be expressed.

82. A proper fraction is strictly part of an integer.

83. Improper fraction is the name given, when an integer or more than an integer is expressed fractionally, as $\frac{7}{7}$, $\frac{18}{7}$, $\frac{13}{13}$, $\frac{159}{13}$, $\frac{137}{137}$, $\frac{219}{137}$.

84. A mixed number consists of a whole number and a fraction, as $8\frac{1}{3}$, $13\frac{1}{9}$, $171\frac{254}{387}$.

85. A compound fraction is a fraction of a fraction, as $\frac{2}{7}$ of $\frac{1}{3}$ of an integer.

86. A complex fraction is one in which the numerator and denominator are fractions, as $\frac{7\frac{1}{2}}{9\frac{3}{7}}$; less complex forms are $\frac{8}{4\frac{1}{3}}$, $\frac{7\frac{1}{2}}{4}$.

87. To EXPRESS AN INTEGER FRACTIONALLY, AND TO CHANGE A MIXED NUMBER TO AN IMPROPER FRACTION, multiply integers by the number indicated by the Table of Fractions (79). In the case of a mixed number, change integers to parts like those given, and add.

Express 18 in sevenths, and reduce $15\frac{1}{3}$ to an improper fraction.

7 *sevenths* = 1 *int.* (79)	3 *thirds* = 1 *int.* (79).
18 ints.	15 int.
7	3
126 sevenths.	45 thirds.
$18 = \frac{126}{7}$	$15\frac{1}{3} = \frac{45}{3} + \frac{1}{3} = \frac{46}{3}$.

Ex. XLVI.

1. Express 9 and 27 in sevenths and elevenths.

2. Express 37 and 127 as fractions, having for their denominators 13 and 17.

3. Reduce $5\frac{1}{8}$, $8\frac{2}{13}$, $13\frac{1}{15}$, $21\frac{1}{10}$ to improper fractions.
4. Reduce $21\frac{1}{3}$, $37\frac{5}{12}$, $73\frac{2}{9}$, $112\frac{1}{11}$ to improper fractions.
5. Express $112\frac{3}{5}$ in fifths, and 39 in fourths.
6 How many tenths are there in 127?

88. TO REDUCE AN IMPROPER FRACTION TO A WHOLE OR MIXED NUMBER, divide the numerator by the number indicated by the Table of Fractions. 79.

5 fifths = 1 int. (79).
No. of int^s is *one-fifth*
of No. of fifths. (38).

Express $\frac{18}{5}$ as a mixed No.
$5\underline{)18}$ fifths.
3 int. 3 fifths i.e. $\frac{18}{5} = 3\frac{3}{5}$.

Ex. XLVII. Express as whole or mixed numbers—

1. $\frac{14}{3}$
2. $\frac{38}{5}$
3. $\frac{211}{12}$
4. $\frac{1713}{11}$
5. $\frac{3179}{9}$
6. $\frac{813}{7}$
7. $\frac{2159}{17}$
8. $\frac{2138}{37}$
9. $\frac{15128}{19}$
10. $\frac{21147}{117}$
11. $\frac{135218}{711}$
12. $\frac{151268}{999}$

89. TO REDUCE A COMPOUND FRACTION TO A SIMPLE ONE, multiply together the numerators of the given fractions for the numerator of the simple fraction required, and the denominators for its denominator.

Express as a simple fraction $\frac{3}{5}$ of $\frac{4}{7}$, i.e. find three-fifths of $\frac{4}{7}$.

(By 81*b*.) $\frac{4}{7}$ may be considered as $7\underline{)\,4}$; taking ✱ to represent the result of the division of 4 by 7, we have to find three-fifths of that result. Dividing by 5 will give one-fifth, which, multiplied by 3, will give the three-fifths required.

7) 4
5) ✱
one fifth of ✱
3
three fifths of ✱

by taking the mult^n first.

4
3
35 | 7) 12
5)

$\frac{3}{5} \times \frac{4}{7} = \frac{12}{35}$

Express as a simple fraction $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{8}$.

8) 7 (81*b*).
4) to find three-fourths
divide by 4
mult. by 3
3
3) to find two-thirds
divide by 3
mult. by 2
2

7
3
21
2
8) 42
96 | 4)
3)

$\frac{2}{3} \times \frac{3}{4} \times \frac{7}{8} = \frac{42}{96}$

But since to multiply and divide by the same factor does not make any real change, it shortens the work to strike out factors which occur both as multipliers and divisors, and also gives the answer in its lowest terms.

$$\frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{4} \times \frac{7}{8} = \frac{7}{16}$$

$$\cancel{2} \times 4$$

90. The striking out Factors, which occur both as Multipliers and Divisors is called Cancelling.

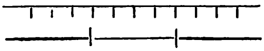
Ex. XLVIII. Express as simple fractions

1. $\frac{1}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$. 2. $\frac{2}{7}$ of $\frac{3}{8}$ of $\frac{1}{12}$. 3. $\frac{4}{9}$ of $\frac{18}{19}$ of 5.
4. $\frac{2}{3}$ of $\frac{1}{7}$ of $2\frac{1}{3}$. 5. $\frac{3}{8}$ of $\frac{3}{5}$ of 3. 6. $\frac{1}{7}$ of $2\frac{1}{3}$ of $3\frac{1}{9}$.
7. $\frac{1}{11}$ of $\frac{5}{12}$ of 12. 8. $\frac{3}{16}$ of $1\frac{7}{9}$ of $9\frac{1}{9}$. 9. $3\frac{1}{4}$ of $2\frac{1}{7}$ of $5\frac{1}{8}$.
10. $4\frac{1}{4}$ of $3\frac{1}{8}$ of $\frac{1}{17}$. 11. $\frac{5}{17}$ of $12\frac{1}{8}$ of 11. 12. $2\frac{1}{6}$ of $8\frac{1}{8}$ of $2\frac{1}{9}$.
13. $\frac{4}{5}$ of 9 of $3\frac{1}{8}$ of 7. 14. $2\frac{1}{12}$ of $3\frac{1}{7}$ of $\frac{1}{5}$ of $5\frac{1}{3}$.
15. $1\frac{1}{9}$ of $1\frac{1}{8}$ of $\frac{1}{13}$ of 3. 16. $3\frac{1}{7}$ of $4\frac{1}{8}$ of $1\frac{1}{5}$ of $5\frac{1}{6}$.

91. To reduce a fraction to its lowest terms, that is, to find an equivalent in which the *parts* into which the integer is divided are as *few as possible, and* therefore as *large as possible:* divide the numerator and denominator, by some number which is a common measure or divisor of them both.

Reduce $\frac{8}{12}$ to its lowest terms.

$$\frac{4)\ 8}{4)\ 12} = \frac{2}{3}$$



In this case the integer is divided into 12 equal parts, and to have the division into 3 equal parts instead of 12, is to have parts four times as large as before; hence there will be only $\frac{1}{4}$th of the number 8, that is 2.

Reduce $\frac{252}{588}$ to its lowest terms.

$$\frac{3)\ 252}{3)\ 588} = \frac{7)\ 84}{7)\ 196} = \frac{4)\ 12}{4)\ 28} = \frac{3}{7}$$

In this example the following changes are made:—

*Denom*rs.	to parts 3 times as large,	7 times as large,	4 times as large
*Numer*s.	to $\frac{1}{3}$ of the No.,	$\frac{1}{7}$th of the No.,	$\frac{1}{4}$th of the No.

Ex. XLIX. Reduce to their lowest terms

1. $\frac{648}{1440}$. 2. $\frac{360}{1728}$. 3. $\frac{648}{396}$. 4. $\frac{792}{2970}$.
5. $\frac{1000}{3375}$. 6. $\frac{2175}{3275}$. 7. $\frac{990}{2420}$. 8. $\frac{452}{576}$.
9. $\frac{3744}{4032}$. 10. $\frac{790}{1835}$. 11. $\frac{4000}{12125}$. 12. $\frac{7776}{10368}$.
13 $\frac{2970}{4320}$. 14. $\frac{1728}{6144}$. 15. $\frac{6500}{8470}$. 16. $\frac{44352}{46656}$.

92. When a common factor of the numerator and denominator cannot be found by sight, the Greatest Common Measure must be worked out (71), and numerator and denominator must be divided by it.

Reduce to its lowest terms $\frac{1576}{1773}$.

1576) 1773 (1
1576
197) 1576 (8
1576

$$\frac{197)\,1576}{197)\,1773} = \frac{8}{9}$$

Ex. L. Reduce to their lowest terms

1. $\frac{85}{153}$. 2. $\frac{1062}{1298}$. 3. $\frac{602}{2709}$.
4. $\frac{573}{2101}$. 5. $\frac{959}{1233}$. 6. $\frac{1904}{1921}$.
7. $\frac{477}{583}$. 8. $\frac{999}{1221}$. 9. $\frac{1415}{3113}$.
10. $\frac{4868}{13387}$. 11. $\frac{24339}{26637}$. 12. $\frac{51597}{120393}$.

93. The *form of a fraction* may be *altered without* any *change of* its *value* by multiplying the numerator and denominator by the same number, that is, by bringing about changes exactly the reverse of those made (91) in reducing a fraction to its lowest terms. In each case the *fraction* is *multiplied by* some *equivalent of* 1, as $\frac{2}{2}$, $\frac{3}{3}$, &c.

$$\frac{1 \times 2}{2 \times 2} = \frac{2 \times 3}{4 \times 3} = \frac{6 \times 4}{8 \times 4} = \frac{24}{32}$$

This change of form is used *to bring Fractions to a common denomr.* for Addition and Subtraction.

To reduce fractions to their least common denominator, find the least common multiple of all the denominators (75). EACH DENOMINATOR must be then CHANGED to the common denominator BY MULTIPLICATION, and in each fraction *numerator and denominator* must be *multiptied by* the *same number* (93).

Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{10}$, to a common denominator.

L. C. M. of 2, 3, 4, 10 = 60 (75).
(by 29) 60 = 2 × 30
= 3 × 20
= 4 × 15
= 10 × 6

$$\frac{1}{2} \times \frac{30}{30} = \frac{30}{60} \qquad \frac{2 \times 20}{3 \times 20} = \frac{40}{60}$$

$$\frac{3 \times 15}{4 \times 15} = \frac{45}{60} \qquad \frac{4 \times 6}{10 \times 6} = \frac{24}{60}$$

Ex. LI. Reduce to their least common denominator

1. $\frac{3}{1}\ \frac{2}{5}\ \frac{1}{6}\ \frac{3}{10}$
2. $\frac{1}{6}\ \frac{5}{7}\ \frac{3}{5}\ \frac{2}{3}$
3. $\frac{2}{3}\ \frac{3}{4}\ \frac{4}{5}\ \frac{5}{6}$
4. $\frac{3}{8}\ \frac{4}{9}\ \frac{1}{16}\ \frac{17}{18}$
5. $\frac{3}{4}\ \frac{3}{8}\ \frac{13}{16}\ \frac{29}{32}$
6. $\frac{4}{5}\ \frac{5}{6}\ \frac{7}{8}\ \frac{9}{10}$
7. $\frac{7}{16}\ \frac{5}{18}\ \frac{13}{24}\ \frac{17}{48}$
8. $\frac{1}{3}\ \frac{2}{9}\ \frac{13}{27}\ \frac{5}{81}$
9. $\frac{2}{7}\ \frac{1}{10}\ \frac{7}{12}\ \frac{17}{35}$
10. $\frac{11}{27}\ \frac{13}{24}\ \frac{1}{6}\ \frac{2}{15}\ \frac{1}{9}$
11. $\frac{2}{25}\ \frac{3}{5}\ \frac{1}{25}\ \frac{13}{30}\ \frac{13}{45}$
12. $\frac{3}{7}\ \frac{11}{12}\ \frac{4}{15}\ \frac{8}{27}\ \frac{8}{33}$

ADDITION OF FRACTIONS.

94. Reduce to the same name (least common denominator), and add together the numerators or numbers of parts.

Add together $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$.

L. C. M. of 3, 4, 5, = 60 (75).
(by 29) $60 = 3 \times 20$
$= 4 \times 15$
$= 5 \times 12$

$$\frac{2}{3} + \frac{3}{4} + \frac{4}{5}$$

$$\text{(by 93)}\ \frac{40 + 45 + 48}{60} = \frac{133}{60} = 2\tfrac{13}{16}.$$

Add together $3\frac{2}{5} + 4\frac{9}{10} + 1\frac{1}{12} + 5$.

$$\frac{2}{5} + \frac{9}{10} + \frac{1}{12}\text{(by 93)} = \frac{24 + 54 + 5}{60} = \tfrac{83}{60} = 1\tfrac{23}{60}.$$

Integers $3 + 4 + 1 + 5 + 1\frac{23}{60} = 14\frac{23}{60}$.

95. Mixed numbers may be changed to improper fractions (87), and comp^{d} fract^{s} must be simplified (89) as preparatory work before changing fractions to others having a common denominator.

Ex. LII. Find the value of

1. $\frac{2}{3}+\frac{3}{5}+\frac{1}{3}+\frac{4}{5}$.
2. $\frac{1}{3}+\frac{1}{4}+\frac{7}{8}+\frac{1}{12}$.
3. $\frac{1}{2}+\frac{1}{4}+\frac{5}{8}+\frac{2}{9}$.
4. $\frac{11}{16}+\frac{7}{15}+\frac{3}{20}+\frac{13}{30}$.
5. $\frac{2}{3}+\frac{5}{6}+\frac{5}{9}+\frac{11}{12}$.
6. $\frac{3}{7}+\frac{1}{12}+\frac{7}{16}+\frac{5}{18}$.
7. $\frac{1}{10}+\frac{14}{15}+\frac{2}{5}+\frac{5}{9}$.
8. $\frac{13}{70}+\frac{9}{21}+\frac{1}{7}+\frac{13}{15}$.
9. $1\frac{1}{2}+2\frac{1}{3}+3\frac{1}{4}$.
10. $2\frac{1}{8}+3\frac{5}{6}+\frac{5}{12}+2\frac{4}{7}$.
11. $3\frac{2}{3}+\frac{1}{5}+5\frac{1}{9}+3\frac{2}{7}$.
12. $2\frac{8}{9}+3\frac{11}{12}+4\frac{15}{16}$.
13. $\frac{2}{7}$ of $1\frac{1}{5}+\frac{4}{9}$ of $3\frac{2}{7}$.
14. $\frac{1}{3}$ of $2\frac{1}{7}$ of $5+1\frac{8}{9}$ of $\frac{1}{4}$ of 4.
15. $\frac{4}{7}+2\frac{1}{3}$ of $3\frac{1}{4}$ of $4\frac{1}{5}$.
16. $8\frac{16}{17}+9\frac{15}{16}+1\frac{31}{32}$.
17. $4\frac{2}{7}$ of $9+5\frac{1}{3}+3\frac{1}{7}$ of 10.
18. $2\frac{2}{7}$ of $3\frac{1}{9}+3\frac{1}{9}$ of $2\frac{2}{7}$.
19. $1\frac{3}{4}$ of $\frac{2}{7}+5\frac{1}{6}+\frac{7}{9}$ of $9+3\frac{1}{3}$ of $\frac{1}{7}+5$.
20. $2\frac{2}{7}$ of $5+3\frac{1}{3}+\frac{4}{9}+\frac{2}{7}$ of $14+5\frac{1}{6}+3\frac{1}{12}$.

SUBTRACTION OF FRACTIONS.

96. As preparatory work, express mixed numbers as improper factions (87), and compound fractions as simple ones (89), then reduce to a common denomination (93), and subtract the less numerator from the greater.

From $\frac{8}{9}$ take $\frac{7}{8}$.

L. C. M. = 72
72 = 9 × 8
= 8 × 9

$$\frac{8\times8}{9\times8}=\frac{64}{72}\qquad\frac{7\times9}{8\times9}=\frac{63}{72};\text{ then }\frac{64-63}{72}=\frac{1}{72}$$

From $4\frac{3}{7}$ take $3\frac{2}{5}$.

L. C. M. = 35
35 = 7 × 5
= 5 × 7

$$\frac{31\times5}{7\times5}=\frac{155}{35}\qquad\frac{17\times7}{5\times7}=\frac{119}{35};\text{ then }\frac{155-119}{35}=\frac{36}{35}=1\frac{1}{35}$$

From $\frac{1}{3}$ of $2\frac{1}{2}$ of 3, take $\frac{3}{7}$ of $\frac{1}{3}$ of $\frac{1}{2}$.

$$\frac{1}{3}\times\frac{5}{2}\times\frac{3}{1}=\frac{5}{2}\qquad\qquad\frac{3}{7}\times\frac{1}{3}\times\frac{1}{2}=\frac{1}{14}$$

L. C. M. = 14
14 = 2 × 7
= 14 × 1

$$\frac{5}{2}-\frac{1}{14}=\frac{35-1}{14}=\frac{34}{14}=2\frac{6}{14}=2\frac{3}{7}.$$

Ex. LIII. Find the value of

1. $\frac{11}{12}-\frac{1}{12}$ $\quad\frac{13}{14}-\frac{5}{14}$ $\quad\frac{8}{11}-\frac{3}{11}$ $\quad\frac{1}{3}-\frac{1}{4}$
2. $4\frac{1}{7}-3\frac{1}{3}$ $\quad2\frac{3}{4}-1\frac{1}{18}$ $\quad4-3\frac{1}{7}$ $\quad10\frac{1}{5}-\frac{11}{30}$
3. $1\frac{1}{23}-\frac{3}{4}$ $\quad8-\frac{16}{17}$ $\quad88\frac{1}{4}-70\frac{1}{2}$ $\quad3\frac{3}{4}-1\frac{1}{7}$
4. $12\frac{1}{13}-3\frac{3}{26}$ $\quad4\frac{1}{100}-\frac{1}{1000}$ $\quad5\frac{2}{7}-\frac{61}{126}$ $\quad23\frac{1}{23}-21\frac{1}{21}$
5. $3\frac{1}{3}$ of $4\frac{1}{7}-1\frac{1}{5}$ of $2\frac{1}{14}$ $\quad8\frac{1}{2}$ of $100-4\frac{1}{16}$ of $\frac{1}{32}$
6. $3\frac{1}{2}-1\frac{1}{4}$ of $\frac{2}{7}$ of $\frac{3}{5}$ $\quad5\frac{4}{9}$ of $3\frac{5}{24}-14\frac{1}{8}$

97. Since fractions must be alike both for addition and subtraction, it is usual, in any expression in which addition and subtraction occur several times, to reduce *all* the fractions at first to a common denominator; then to find the *sum* of the fractions to be added, and of those to be subtracted, and thus bring the expression to be represented by two fractions only. Mixed numbers and compound fractions necessitate preparatory work.

24 = 3 × 8
= 4 × 6
= 2 × 12
= 12 × 2
= 6 × 4
= 8 × 3

$$2\tfrac{1}{3}-3\tfrac{1}{4}+3\tfrac{1}{2}-4\tfrac{1}{12}+5\tfrac{1}{6}-3\tfrac{1}{8}$$

$$\frac{7}{3}-\frac{13}{4}+\frac{7}{2}-\frac{49}{12}+\frac{31}{6}-\frac{25}{8}$$

L. C. M. = 24.

$$\begin{array}{rr} + & - \\ 56 & 78 \\ 84 & 98 \\ 124 & 75 \\ \hline 264 & 251 \end{array}$$

$$\frac{56-78+84-98+124-75}{24}=\frac{264-251}{24}=\frac{13}{24}$$

Ex. LIV.

1. $3\frac{1}{2} + 5\frac{2}{7} - 3\frac{1}{4} - 5\frac{1}{14} + 2\frac{1}{6} - 1\frac{1}{12}$.
2. $3\frac{1}{2}$ of $5\frac{2}{7} + 2\frac{1}{8}$ of $3\frac{1}{14} - \frac{1}{7}$ of $3\frac{1}{2}$ of $\frac{1}{21}$.
3. $2\frac{5}{6} - 3\frac{3}{10} + 5\frac{1}{5} + \frac{1}{2} + 18\frac{1}{6} - 2\frac{1}{4} - 3\frac{5}{16}$.
4. $8\frac{1}{8} + 2\frac{1}{9} + 5\frac{1}{4} - 3\frac{1}{24} - 1\frac{1}{12} - 2\frac{3}{16}$.
5. $4\frac{1}{2}$ of $6 + 3\frac{2}{3}$ of $9 - 8\frac{1}{8}$ of $\frac{1}{16} - \frac{1}{4}$ of $\frac{1}{3}$.
6. $4\frac{2}{9} - 5\frac{2}{3} + 3\frac{1}{27} - 1\frac{1}{6} + 1\frac{1}{8} - \frac{1}{16}$.

MULTIPLICATION OF FRACTIONS.

98. Multiply together the numerators of the given fractions, also the denominators. Cancel (90), at first, to get the product as a fraction in its lowest terms.

Multiply $\frac{3}{4}$ by $\frac{5}{7}$.

$\frac{15}{4}$ i.e $4\underline{)15}$

$4\underline{)15}$
$7\underline{)}$

$28\underline{)15}$

Multiply* 3 fourths ($\frac{3}{4}$) by 5, we get 15 fourths ($\frac{15}{4}$); but the multiplier given is not 5, but one-seventh of 5 (81 *b*); hence our result—15 fourths—is seven times too great; taking then one-seventh of that result, we obtain the result required.

$$\frac{3}{4} \times \frac{5}{7} = \frac{15}{28}$$

99. It will be seen that in Multiplication of Fractions, the method of working is the same as that by which a Compound Fraction is brought to a simple one (89).

Ex. LV.

1. $\frac{3}{8} \times \frac{4}{7} \times 2\frac{1}{3}$; $3\frac{1}{7} \times 2\frac{1}{9} \times 18$; $4\frac{3}{8} \times 3\frac{2}{7} \times 1\frac{4}{5}$.
2. $8\frac{1}{7} \times 3\frac{1}{2} \times 8\frac{1}{4}$; $4\frac{2}{7} \times 5\frac{2}{7} \times 1\frac{1}{8}$; $3\frac{3}{4} \times 3\frac{3}{4} \times \frac{8}{15}$.
3. $2\frac{1}{2}$ of $4\frac{1}{5} \times 3\frac{1}{8}$; $1\frac{1}{4} \times 2\frac{2}{9} \times 3\frac{7}{11}$; $5\frac{1}{3} \times 2\frac{1}{8} \times 3\frac{1}{6}$.
4. $\frac{1}{7}$ of $\frac{11}{12} \times 8\frac{1}{9}$; $4\frac{7}{16} \times 3\frac{1}{3} \times 1\frac{1}{7}$; $1\frac{1}{2} \times 3\frac{1}{5} \times 8\frac{1}{12}$.
5. $\frac{2}{3}$ of $1\frac{2}{3} \times 1\frac{7}{9}$ of $3\frac{2}{7}$; $2\frac{2}{3}$ of $3\frac{2}{7}$ of $5\frac{1}{2} \times 2\frac{1}{7}$ of $1\frac{1}{8} \times \frac{8}{9}$.

DIVISION OF FRACTIONS.

100. Having changed mixed numbers to improper fractions, and compound fractions to simple ones, invert the divisor and multiply.

$$20\left|\begin{array}{l}4)\underline{3\quad}\\ 5)\underline{\quad\;}\end{array}\right.$$

$$20\left|\begin{array}{l}4)\underline{3\quad}\\ 5)\underline{\quad\;}\end{array}\right.$$

$$\underline{\qquad 7}$$

$$\begin{array}{r}3\\ \underline{7}\end{array}$$

$$20\left|\begin{array}{l}4)\underline{21}\\ 5)\underline{\quad\;}\end{array}\right.$$

mult^n first.

Divide $\frac{3}{4}$ by $\frac{5}{7}$.

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4)$\underline{3}$ divided by 5 gives us as a result 20)$\underline{3}$; but the given divisor is not 5, but one-seventh of 5; our divisor 5 is therefore seven times too great, and our quotient is consequently one-seventh only of what it should be—hence making it seven times as great as it is, will bring the required result.

$$\frac{3}{4} \div \frac{5}{7} = \frac{3}{4} \times \frac{7}{5} = \frac{21}{20} = 1\tfrac{1}{20}.$$

Divide $3\frac{1}{3}$ of $1\frac{1}{4}$ of $1\frac{1}{6}$ by $4\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{7}$.

$$\frac{\cancel{10}}{3} \times \frac{5}{4} \times \frac{7}{\cancel{6}} = \frac{175}{36} \quad\Big|\quad \frac{9}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{7} = \frac{7}{9}.$$

$$\frac{175}{39} \div \frac{9}{7} = \frac{175}{36} \times \frac{7}{9} = \frac{1225}{324} = 3\tfrac{253}{324}.$$

Simplify $\dfrac{1\frac{1}{4} \times \frac{2}{5} \times 1\frac{1}{8}}{3\frac{1}{6} - \frac{4}{5}\left(1 - \dfrac{1}{5\frac{1}{2}}\right)}$.

In this example the division will consist in dividing the numerator by the denominator (81), but both must first be simplified.

The numerator in which add^n and subt^n occur must be worked out by (97).

$$\frac{5}{4} + \frac{2}{5} - \frac{9}{8}; \quad \text{L. C. M.} = 40 \quad \frac{50 + 16 - 45}{40} = \frac{21}{40}\ \textit{num}^r.$$

The several parts of the denominator will be worked thus :—

$$\text{div}^n\ \frac{1}{5\frac{1}{2}} = \frac{1}{1} \div \frac{11}{2} = \frac{1}{1} \times \frac{2}{11} = \frac{2}{11} \;\Big|\; \text{sub}^n\ 1 - \frac{2}{11} = \frac{11}{11} - \frac{2}{11} = \frac{9}{11}.$$

$$\text{mult}^n\ \frac{9}{11} \times \frac{4}{5} = \frac{36}{55} \;\Big|\; \text{sub}^n\ 3\tfrac{1}{6} - \frac{36}{55} = \frac{19}{6} - \frac{36}{35} = \frac{665 - 216}{210} = \frac{449}{210}\ \textit{denom}^r.$$

$$\text{division}\ \frac{21}{40} \div \frac{449}{210} = \frac{21}{4\cancel{0}} \times \frac{21\cancel{0}}{449} = \frac{441}{1796}$$

101. In simplifying a fractional expression, it must be remembered that *Multiplication and Division take precedence of Addition and Subtraction*, because they bring about respectively the change of Compound and Complex Fractions to simple ones.

Ex. LVI. Simplify

1. $\frac{3}{4} \div \frac{5}{6}$; $\frac{2}{3} \div \frac{8}{4}$; $4\frac{1}{2} \div 5\frac{1}{3}$; $6\frac{1}{9} \div \frac{8}{15}$.
2. $9 \div 3\frac{1}{7}$; $3\frac{1}{7} \div 9$; $10 \div 5\frac{1}{2}$; $5\frac{1}{2} \div 10$.
3. $\frac{2}{3}$ of $3\frac{1}{2} \div \frac{2}{27}$; $4\frac{1}{5}$ of $2\frac{1}{9} \div 3\frac{3}{4}$ of $1\frac{1}{8}$; $2\frac{2}{7}$ of $5\frac{1}{3} \div 1\frac{1}{6}$ of $\frac{1}{8}$.
4. $\frac{7\frac{1}{2}}{3\frac{1}{5}}$; $\frac{2\frac{1}{4}}{9}$; $\frac{9}{2\frac{1}{4}}$; $\frac{11\frac{1}{2}}{8\frac{1}{5}}$; $\frac{7}{8\frac{1}{4}}$.
5. $\frac{2\frac{2}{3} + \frac{2}{5}}{2\frac{2}{3} - \frac{2}{5}}$; $\frac{3\frac{1}{2} \text{ of } 2\frac{1}{4}}{1\frac{1}{3} \text{ of } 1\frac{1}{4}}$; $\frac{2\frac{1}{4} \text{ of } 5\frac{1}{2}}{2\frac{1}{2} + 1\frac{1}{3}}$; $\frac{2\frac{1}{2} - 1\frac{1}{3}}{2\frac{1}{4} \text{ of } 5\frac{1}{2}}$.
6. $\dfrac{1\frac{1}{2} + 3\frac{2}{7} - 1\frac{1}{3}}{2\frac{1}{3} + \left(3 - \dfrac{1}{6\frac{2}{3}}\right)}$; $\dfrac{1\frac{1}{2} \text{ of } 3\frac{2}{7} \text{ of } 1\frac{1}{3}}{2\frac{1}{3} \text{ of } \frac{3}{7} \text{ of } \left(\dfrac{1}{4\frac{1}{2}} + \dfrac{1}{6\frac{1}{2}}\right)}$.
7. $\frac{5\frac{1}{2}}{7\frac{1}{3}} + \frac{2\frac{3}{5}}{3\frac{1}{8}}$; $\frac{1\frac{1}{4} + 2\frac{1}{5}}{3\frac{1}{2} - 1\frac{1}{8}}$.
8. $\frac{\frac{1}{3} + \frac{1}{4} + \frac{1}{7} + \frac{1}{14} + \frac{1}{28}}{\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{13}{14} + \frac{1}{28}}$; $\frac{3\frac{3}{5} + 4\frac{1}{7}}{3\frac{1}{9} - 2\frac{1}{4}}$.

102. TO FIND WHAT FRACTION OR PART ONE QUANTITY IS OF ANOTHER, bring both to the same name, and find the fraction according to the method indicated by the following example.

(*a.*) What part of 18*s.* 6*d.* is 3*s.* 6*d*?

18*s.* 6*d.* = 37 sixpences. | 1 sixpence = $\frac{1}{37}$ part of 37 sixpences.

3*s.* 6*d.* = 7 „ | 7 sixpences = $\frac{7}{37}$ „ „

(*b.*) Express £$\frac{2}{3}$ as a fraction of $\frac{4}{5}$ of a guinea.

$$\frac{£2}{3} \times 20 = \frac{40s.}{3} \qquad \frac{\frac{40}{3}}{\frac{84}{5}} = \frac{\overset{10}{\cancel{40}}}{3} \times \frac{5}{\underset{21}{\cancel{84}}} = \frac{50}{63}$$

$$\text{Gui. } \frac{4}{5} \times 21 = \frac{84s.}{5}$$

Ex. LVII. Reduce

1. 4*s.* 7*d.* to the fr. of £2; 3*s.* 8$\frac{1}{2}$*d.* to the fr. of 4*s.* 6*d.*

2. £3 4*s.* 9*d.* to the fr. of £14 5*s.* 6*d.*; 5*s.* 10$\frac{1}{2}$*d.* to the fr. of 1 guinea.

3. 2 qrs. 15 lbs. to the fr. of 4 cwt. 1 qr.; 1 qr. 8 lbs. to the fr. of 1 cwt.

4. 1 ton 1 cwt. 1 qr. to the fr. of 15 tons; 4 lbs. 5 oz. 15 drs. to the fr. of 1 qr.

5. £4 7*s.* 10*d.* to the fr. of £9 4*s.* 8*d.*; £14 15*s.* to the fr. of 137 crowns.

6. 1 a. 1 r. 18 p. to the fr. of 5 acres; £18 to the fr. of 18 guineas.

7. 1 cwt. 1 qr. to the fr. of 3 qrs. 5 lbs.; 7 w. 1 d. 9 hrs. to the fr. of 31 days.

8. 2 w. 4 d. 13 hrs. to the fr. of 4 wks.; 1 w. 1 d. 1 hr. to the fr. of 1 d. 9 hrs.

9. 4 a. 2 r. 15 p. to the fr. of 100 acres; £4 9*s.* 7$\frac{1}{2}$*d.* to the fr. of £5 9*s.* 8$\frac{1}{4}$*d.*

10. 2 sq. yds. 1 sq. ft. 187 sq. in. to the fr. of 19 sq. yds.; 2 m. 1 f. 8 p. to the fr. of 5 miles.

11. £1$\frac{1}{2}$ to the fr. of 1$\frac{1}{2}$ guineas; 3$\frac{1}{2}$ crs. to the fr. of £$\frac{1}{2}$.

12. 2$\frac{3}{4}$*s.* to the fr. of £1; $\frac{3}{5}$ gui. to the fr. of $\frac{3}{7}$*s.*

13. $\frac{2}{3}$*d.* to the fr. of £1; 2$\frac{4}{9}$ cwts. to the fr. of 1 ton.

14. 3$\frac{1}{2}$ of 2$\frac{1}{3}$*s.* to the fr. of 7$\frac{1}{2}$*s.*; $\frac{5}{7}$ of 2$\frac{1}{2}$*d.* to the fr. of 1*s.*

EXAMPLES OF ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION OF FRACTIONS OF CONCRETE QUANTITIES.

Ex. LVIII.

	£	*s.*	*d.*	
1.	2	4	6$\frac{1}{4}$	= $\frac{3}{12}$
	3	5	8$\frac{1}{3}$	= $\frac{4}{12}$
	4	8	2$\frac{1}{2}$	= $\frac{6}{12}$
	2	12	5$\frac{1}{12}$	= $\frac{1}{12}$
	3	19	8$\frac{5}{6}$	= $\frac{10}{12}$
	2	17	10$\frac{1}{4}$	= $\frac{3}{12}$

	£	*s.*	*d.*
2.	7	5	8$\frac{3}{7}$
	2	4	10$\frac{1}{14}$
	5	19	11$\frac{2}{7}$
	8	13	6$\frac{1}{2}$
	4	18	8$\frac{1}{7}$
	5	4	6$\frac{3}{14}$

	£	*s.*	*d.*
3.	2	7	9$\frac{1}{3}$
	5	8	2$\frac{1}{8}$
	4	18	10$\frac{5}{6}$
	3	14	9$\frac{7}{12}$
	5	12	8$\frac{3}{4}$
	2	17	7$\frac{2}{3}$

4.	8	6	9$\frac{2}{5}$
	4	8	8$\frac{1}{10}$
	3	17	7$\frac{19}{20}$
	8	18	2$\frac{3}{5}$
	4	4	6$\frac{3}{10}$
	7	17	5$\frac{1}{5}$

5.	8	6	8$\frac{1}{8}$
	4	3	10$\frac{1}{16}$
	2	7	9$\frac{1}{4}$
	8	5	2$\frac{3}{4}$
	4	8	6$\frac{3}{16}$
	5	2	9$\frac{3}{8}$

6.	4	8	5$\frac{1}{2}$
	3	17	8$\frac{1}{4}$
	2	19	6$\frac{5}{8}$
	4	17	2$\frac{1}{8}$
	3	5	8$\frac{2}{8}$
	2	4	2$\frac{3}{8}$

	£	s.	d.		£	s.	d.		£	s.	d.
7.	17	18	$6\frac{1}{3}$	8.	5	6	$8\frac{7}{8}$	9.	17	3	$6\frac{3}{4}$
	24	5	$8\frac{1}{4}$		2	14	$5\frac{1}{4}$		17	3	$6\frac{3}{8}$
	14	14	$9\frac{1}{3}$		3	18	$6\frac{3}{8}$		14	5	$2\frac{1}{2}$
	27	5	$6\frac{3}{4}$		4	15	$2\frac{3}{4}$		15	8	$7\frac{1}{8}$
	18	8	$4\frac{1}{12}$		18	19	$11\frac{1}{8}$		14	3	$9\frac{5}{8}$
	17	5	$9\frac{11}{12}$		45	4	$6\frac{1}{4}$		17	2	$6\frac{1}{4}$

Ex. LIX.

		£	s.	d.		£	s.	d.		£	s.	d.	
1.	From	14	0	$6\frac{1}{3} = \frac{2}{6}$ (93)	00	0^{20}	0^{12}	$\frac{0}{6}\,{}^{\frac{6}{6}}$	14	0^{20}	6^{12}	$\frac{2}{6}\,{}^{\frac{6}{6}}$	
	Take	7	8	$7\frac{1}{2} = \frac{3}{6}$ (93)		0^{1}	0^{1}	0^{1} $\frac{0}{6}$		7^{1}	8^{1}	7^{1} $\frac{3}{6}$	

2. 3 14 $8\frac{1}{8} = \frac{1}{8}$ add to upper line $\frac{8}{8}$*d.* and 12*d.*
2 3 $11\frac{1}{4} = \frac{2}{8}$ (93) „ lower „ 1*d.* and 1*s.*

	£	s.	d.		£	s.	d.		£	s.	d.
3.	5	16	$7\frac{1}{6}$	4.	16	0	$3\frac{5}{8}$	5.	3	17	$7\frac{7}{12}$
	2	19	$8\frac{1}{8}$		4	17	$8\frac{5}{12}$		1	18	$9\frac{5}{16}$
6.	7	14	$8\frac{3}{4}$	7.	5	16	$2\frac{1}{8}$	8.	3	2	$6\frac{1}{2}$
	3	15	$9\frac{2}{3}$		3	14	$6\frac{1}{2}$		2	15	$8\frac{5}{8}$

103. (*a*) Find the value of $\frac{3}{4}$ of £6 2*s.* 8*d.*

£ s. *d.*		£ s. *d.*
4) 6 2 8 (1 time).	Worked thus—	6 2 8
one-fourth	*first mult*^{n}., then div^n.	3
3	since $\frac{1}{4}$ of 3 = $\frac{3}{4}$ of 1.	4) three times
three-fourths		$\frac{1}{4}$ of 3 times.

(*b*) Find the value of $\frac{5}{8}$ of 1 guinea.

£ s. *d.*
8) 5 5 0
13 $1\frac{1}{2}$

Whenever possible, the mult^n. should be done mentally, as in this example.

(*c*) Find the value of $\frac{7}{9}$ of 10*s.* 6*d.* − $\frac{3}{4}$ of 2*s.* 6*d.* + £$\frac{1}{12}$ − $\frac{1}{14}$ of a guinea.

s. d.	s. d.	s. d.	s. d.
9) 73 6	4) 7 6	12) 20 0	14) 21 0
8 2	1 $10\frac{1}{2}$	1 8	1 6

8*s.* 2*d.* + 1*s.* 8*d.* − 1*s.* $10\frac{1}{2}$*d.* − 1*s.* 6*d.* = 6*s.* $5\frac{1}{2}$*d.*

(*d*) Multiply £3 10*s*. 8$\frac{5}{8}$*d*. by 5$\frac{3}{7}$.

$$\begin{array}{rrl} \pounds & s. & d. \\ 3 & 10 & 8\frac{5}{8} \\ & & 5 \\ \hline 17 & 13 & 7\frac{1}{8} = \frac{7}{56}\ (93) \\ 1 & 10 & 3\frac{39}{56} \\ \hline 19 & 3 & 10\frac{46}{56} \end{array} \qquad \begin{array}{lrrl} & \pounds & s. & d. \\ \frac{3}{7} \text{ of} & 3 & 10 & 8\frac{5}{8} \\ & & & 3 \\ \hline 7) & 10 & 12 & 1\frac{7}{8} \\ \hline & 1 & 10 & 3\frac{39}{56} \end{array} \qquad \begin{array}{l} 4\frac{7}{8} = \frac{32}{8} + 7 \\ \quad = \frac{39}{8} \times \frac{1}{7} = \frac{39}{56} \end{array}$$

(*e*) Divide 8*s*. 1$\frac{1}{2}$*d*. by $\frac{2}{113}$.

$$97\tfrac{1}{2}d. = \frac{195d.}{2} \times \frac{113}{2} = \frac{22035d.}{4} = 5508\tfrac{3}{4}d. = \pounds 22\ \ 19\ \ 0\tfrac{3}{4}.$$

Ex. LX. Find the value of

1. $\frac{3}{5}$ of £1; $\frac{7}{20}$ of £5; $\frac{2}{3}$ of 6*s*. 8*d*.; 2$\frac{8}{9}$ of £1.
2. £2 5*s*. 4*d*. × $\frac{7}{10}$; £4 6*s*. 8$\frac{1}{3}$*d*. × 1$\frac{1}{2}$; £8 14*s*. 9$\frac{3}{5}$*d*. × 1$\frac{2}{5}$.
3. £2 5*s*. 4*d*. ÷ $\frac{7}{10}$; £4 6*s*. 8$\frac{1}{2}$*d*. ÷ 1$\frac{1}{2}$; £8 14*s*. 9$\frac{2}{7}$*d*. ÷ 1$\frac{2}{5}$.
4. £18 2*s*. 7$\frac{1}{2}$*d*. × 3$\frac{1}{8}$; £15 3*s*. 7$\frac{3}{4}$*d*. × 2$\frac{5}{8}$; £5 8*s*. 6$\frac{3}{4}$*d*. × 3$\frac{3}{4}$.
5. £18 18*s*. 5$\frac{1}{2}$*d*. ÷ $\frac{4}{25}$; £9 17*s*. 6*d*. ÷ $\frac{3}{77}$; £8 2*s*. 9$\frac{3}{4}$*d*. ÷ $\frac{12}{49}$.
6. $\frac{3}{7}$ of a ton; $\frac{3}{7}$ of a lb. Troy; 3$\frac{1}{4}$ cwt. × 1$\frac{1}{3}$.
7. 3 w. 1 d. × $\frac{9}{32}$; 3 w. 1 d. ÷ $\frac{9}{32}$; 2*s*. 7$\frac{3}{4}$*d*. × $\frac{1}{2}$ of 2$\frac{3}{4}$.
8. $\frac{3}{8}$ of 18$\frac{1}{3}$*s*.; 2 c. 1 qr. 5 lb. × 5$\frac{5}{24}$; 12$\frac{4}{7}$ of £3 18*s*. 6*d*.
9. £1 5*s*. 8*d*. × $\frac{2\frac{1}{2}}{3\frac{1}{3}}$; $\frac{1\frac{1}{8}}{3\frac{1}{5}}$ of £3 7*s*. 6*d*.; $\frac{2\frac{1}{3}}{3\frac{1}{4}}$ of $\frac{1\frac{1}{2}}{2\frac{1}{3}}$ of 3$\frac{1}{2}$ guineas.
10. £2$\frac{5}{8}$ + 3$\frac{1}{8}$*s*. + 4$\frac{1}{8}$*d*.; £$\frac{5}{8}$ − $\frac{3}{8}$*s*. + 9$\frac{3}{4}$*d*.; £1$\frac{5}{6}$ + 3$\frac{1}{3}$*s*. + 8$\frac{1}{2}$*d*.
11. 1$\frac{3}{4}$ of 10*s*. 6*d*. + $\frac{3}{8}$ of 5*s*. + £$\frac{1}{8}$ − $\frac{1}{8}$*s*.; $\frac{3}{8}$ of 21*s*. + $\frac{1}{3}$ of 20*s*. − $\frac{1}{4}$ of 7*s*. 6*d*.
12. $\frac{3}{4}$ of 21*s*. + $\frac{3}{8}$ of £2 − $\frac{1}{12}$ of £$\frac{1}{2}$; $\frac{3}{7}$ of £13 + 2$\frac{2}{7}$ of £1 + $\frac{1}{3}$ of $\frac{2}{3}$ of $\frac{5}{9}$ of 18*s*.

MISCELLANEOUS EXAMPLES (XXXVII—LX).

1. Reduce $\frac{3}{10}$, $\frac{4}{15}$, $\frac{8}{25}$, to a common denominator. 93.
2. Find the sum of $\frac{3}{10}$, $\frac{4}{15}$, $\frac{8}{25}$. 94.
3. Find the product of $\frac{3}{10}$, $\frac{4}{15}$, $\frac{8}{25}$. 98.
4. Find the L. C. M. of 2, 3, 4, 5, 6, 7, 8. 75.
5. Find the G. C. M. of 114, 178, and 216. 72.
6. Simplify 13$\frac{1}{3}$ − 2$\frac{1}{4}$ + 5$\frac{1}{8}$ − 6$\frac{1}{2}$ − 1$\frac{1}{4}$. 97.
7. Divide $\frac{4\frac{1}{2}}{7\frac{1}{2}}$ of $\frac{2\frac{1}{8}}{1\frac{1}{4}}$ by $\frac{3}{7}$ of 4$\frac{1}{2}$ of $\frac{1}{1\frac{1}{8}}$. 100.
8. Find the value of $\frac{4}{7}$ of a ton − $\frac{3}{8}$ cwt. 103 *c*.
9. Mult^y^ £4 3*s*. 8$\frac{1}{3}$*d*. by 4$\frac{2}{7}$. 103 *d*.
10. Divide £3 1*s*. 6$\frac{2}{3}$*d*. by $\frac{117}{5}$. 103 *e*.

11. Which is the greatest and which the least of $\frac{3}{10}$, $\frac{4}{5}$, $\frac{9}{125}$. 93.

12. Find the sum of $\frac{3}{5}$ of £2 17*s.* 6*d.*, and $\frac{7}{9}$ of $4\frac{1}{2}$ guineas. 103 *c.*

13. Which is the greater $\frac{3}{8}$ of $2\frac{2}{9}$ or $\frac{11}{12}$ of $1\frac{1}{3}$, and by how much? 89, 93, 96.

14. Find the value of $\frac{2\frac{1}{2}}{3\frac{1}{2}}$ of $3\frac{1}{2}$ guineas less $\frac{1}{12}$ of $\frac{1}{8}$ of £10. 103 *c.*

15. Multʸ £2 6*s.* $7\frac{3}{5}$*d.* by $7\frac{3}{8}$. 103 *d.*

16. Divide £2 6*s.* $7\frac{3}{5}$*d.* by $7\frac{3}{8}$. 100, 103 *e.*

17. What part of 17*s.* 9*d.* is 3*s.* $8\frac{1}{2}$*d.* 102.

18. Find the sum of £4 8*s.* $6\frac{1}{4}$*d.*; £2 3*s.* $8\frac{1}{5}$*d.*; £5 2*s.* $9\frac{1}{3}$*d.*; and £13 $4\frac{1}{2}$*s.* 103.

19. From £181 7*s.* $10\frac{1}{3}$*d.* take £99 13*s.* $6\frac{1}{2}$*d.* Ex. LIX.

20. Reduce 3 qrs. 17 lb. to the fraction of 4 c. 1 qr. 9 lbs. 102.

21. The greater of two fractions is $11\frac{1}{7}$, their sum is $19\frac{1}{8}$, find the less. 96.

22. The less of two fractions is $6\frac{3}{11}$, and their sum is $25\frac{3}{8}$, find the greater. 96.

23. The product is $171\frac{2}{13}$, and the multiplicand $13\frac{1}{5}$, find the multiplier. 33 *c.*

24. The multiplier is $18\frac{1}{9}$, and the product $132\frac{5}{7}$, find the multiplicand. 33 *b.*

25. The dividend is $183\frac{11}{17}$, the quotient $1\frac{1}{17}$, find the divisor. 34 *b.*

26. The quotient is $144\frac{2}{13}$, the divisor $14\frac{1}{9}$, find the dividend. 34 *c.*

27. The quotient is $18\frac{1}{3}$, the divisor $7\frac{1}{8}$, and the remainder $3\frac{1}{5}$, find the dividend. 34 *c*, second case.

28. The subtrahend is $\frac{2}{3}$ of $\frac{3}{7}$ of 5, and the remainder $\frac{1\frac{1}{9}}{3\frac{1}{3}}$, find the minuend. 15 *c.*

29. What number added to $3\frac{1}{4}$ will make $16\frac{2}{9}$? 96.

30. What number taken from $3\frac{1}{4}$ will leave $1\frac{7}{8}$? 96.

31. The length of a room being $18\frac{3}{4}$ ft., and the width $12\frac{5}{9}$ ft., find the number of square yards of floor? 64, *a*, *b.*

32. The surface of a wall measures 260 sq. ft., and its height is $16\frac{1}{4}$ ft., find the length. 65 *b.*

33. The four walls of a room measure $19\frac{3}{8}$, $19\frac{3}{8}$, $18\frac{1}{4}$, $18\frac{1}{4}$ ft. in length, and their height is $12\frac{3}{4}$ ft., find the measurement of the surface of the walls. XLI., 4.

34. If the surface of the four walls of a square room measures 776 sq. ft. and the height of the room is $12\frac{1}{8}$ ft., find the length of a side of the room. 65 *b.*

35. The measurements of a block of stone are $13\frac{1}{4}$ ft., $8\frac{1}{4}$ ft., and $3\frac{1}{6}$ ft., find the number of cubic feet of stone. 68.

36. Of a block of stone, containing $318\frac{3}{4}$ cub. ft., the width is $4\frac{1}{4}$ ft., and the thickness $3\frac{1}{8}$ ft., find the length. 69, *a.*

37. Find the simple fraction equal to

$$3\frac{1}{3} - \frac{3}{7} \text{ of } \frac{7}{9} \text{ of } 5 + 23\frac{3}{7} - \frac{1-\frac{1}{2}}{1-\frac{2}{3}}. \quad 97.$$

38. Find the value of $(\frac{1}{2} + \frac{3}{7}) \times (1\frac{1}{2} + 3\frac{1}{4}) \times \frac{7\frac{1}{4}}{8\frac{1}{8}} \times (2\frac{1}{16} - 1\frac{7}{8})$.

39. Find the value of $\frac{2\frac{2}{3} - \frac{2}{3}}{(\frac{1}{3} \text{ of } 5) \div \frac{2}{3}} \times \frac{3}{7} \text{ of } \frac{1\frac{1}{2} \text{ of } 3\frac{1}{4}.}{3\frac{1}{4} + 1\frac{1}{2}.}$ 81 *b.*

40. Find the value of $\frac{\frac{1}{4} + \frac{7}{8} - \frac{1}{3}}{\frac{1}{4\frac{1}{2}} + \frac{1}{2\frac{1}{2}} - \frac{1}{12\frac{1}{2}}}$. 97, 100, 81 *b.*

41. If I give away $\frac{1}{3}$ of my money, then $\frac{1}{4}$ of it, what part still remains? 94, 96.

42. What number of times is $\frac{3}{8}$*s.* contained in £121$\frac{7}{8}$? 59, 100.

43. If $\frac{7}{17}$ of a farm be worth £2000 10$\frac{3}{4}$*s.*, what is the value of $\frac{1}{17}$? 57.

44. After taking from a purse $\frac{3}{4}$ of its contents, it was found that $\frac{1}{3}$ of the money left amounted to 18*s.* 6*d.*; what money did the purse contain?

45. Find the value of $\frac{5\frac{1}{2} + 3\frac{1}{6}}{9\frac{1}{3}} \div \frac{3\frac{1}{4} + 4\frac{1}{3}}{8\frac{3}{14} - 2\frac{1}{7}.}$ 81 *b.*

46. If a pint contains 34$\frac{2}{3}$ cub. in., how many gallons of water can be placed in a cistern measuring 4 ft. 6 in. × 3 ft. 4 in. × 2 ft. 9 in.? 68.

47. What length of carpet, $\frac{5}{8}$ yd. wide, will cover a room 29$\frac{1}{2}$ ft. by 11$\frac{1}{4}$ ft.? 65 *a*, *b.*

48. Express as a fraction of £7 the difference between £4$\frac{5}{7} \div 1\frac{1}{7}$ and £4$\frac{5}{7} \times 1\frac{1}{7}$. 102 *b.*

49. How many yards of paper $\frac{2}{3}$ yd. will be required for the walls of a room that is 16 ft. long, 14 ft. broad, and 12 ft. high, and what will be the cost at 2$\frac{1}{2}$*d.* per yard? XLI. *y.*

50. A cubic foot of wood weighing 10$\frac{3}{7}$ lbs., find the weight of a beam 36 ft. long, 5 ft. 2 in. wide, and 3 ft. 6 in. thick. 68.

DECIMAL FRACTIONS.

104. In Decimal Fractions (Decim-us Latin = tenth) or tenth-parts, the integer or unit is at first divided into 10 equal parts, and in the successive divisions, each part is subdivided into ten equal parts; hence from the divisions into 10, 100, 1000, 10000, 100000, 1000000, &c., equal parts, there are parts named '*tenths*,' '*hundredths*,' '*thousandths*,' '*ten-thousandths*' (tenths of thousandths), '*hundred-thousandths*' (hundredths of thousandths), '*millionths*,' &c.

INTEGERS	•	0	0	0	0	0	0
		tenths.	hundredths.	thousandths.	ten-thousandths.	h^d-thousandths.	millionths.

105. A dot called the Decimal Point is placed between INTEGERS and decimal parts; and in decimals, as in integers (5), cyphers must be used when necessary to keep the figures in the positions which give them their right value.

106. In decimals, as in integers, each group has 10 times the value of that on its right, and $\frac{1}{10}$th of the value of that on its left, and such relative value is uniform from millionths up to MILLIONS, and from MILLIONS down to millionths.

	h	t		h	t								
0	0	0	0	0	0	0	•	0	0	0	0	0	0
M			TH			U		tenths.	hundredths.	thousandths.	ten-thousandths.	h^d-thousandths.	millionths.

Hence to remove figures one place to the left is to give them a ten-fold value, that is, to multiply them by 10; and to move them one place to the right is to give them $\frac{1}{10}$th of their value before removal, that is, to divide by 10, and this is the case both as to integers and decimal parts. In such changes the *decimal point must be regarded as fixed.*

13·125 × 10 becomes 131·25
13·125 ÷ 10 „ 1·3125
13·125 × 100 „ 1312·5
13·125 ÷ 100 „ ·13125

Multiply and divide 2·375 by 10 and by 100 (10 × 10).

2·375 × 10 = 23·75 one move of all figures to the left.
2·375 × 100 = 237·5 two moves " " " "
2·375 ÷ 10 = ·2375 one move " " " right.
2·375 ÷ 100 = ·02375 two moves " " " "

Ex. LXI.

1. Multiply and divide 147·215 by 10.
2. " " 152·5 by 100.
3. " " 1205·1205 by 1000.
4. Multiply ·1325 by 10 and divide the product by 100.
5. Multiply and divide ·5 by 10, also by 100.
6. Multiply ·125 by successive tens until it becomes 125·.
7. How often must ·125 be divided by 10 to make it ·00125.
8. By what number must ·0115 be multiplied to make it a whole number.

107. In decimals, as in integers, and also in any set of figures containing both integers and decimals, the *last figure used gives the name*, thus 15·175 may be taken in parts, as

1 | 5·175 using 1.... is 1 ten, &c.
15 | ·175 " 15... " 15 (units), &c.
15·1 | 75 " 151.. " 151 tenths, &c.
15·17 | 5 " 1517. " 1517 hundredths, &c.
15·175 " 15175 " 15175 thousandths;

and if the units are taken separately

15·1 | 75 is ..... 15 units and 1 tenth, &c.
15·17 | 5 " ..... " and 17 hundredths, &c.
15·175 " ..... " and 175 thousandths.

Express as decimals, 3 tenths, 7 hundredths, 19 millionths.

t$^{\text{ths}}$	h$^{\text{ths}}$	m$^{\text{ths}}$
·0	·00	·000000
·3	·07	·000019

Ex. LXII.

1. Express as decimals $\frac{1}{100}$, $\frac{7}{10}$, $\frac{113}{1000}$, $\frac{11}{1000000}$.
2. " " 3 tenths, 103 tenths, 17 millionths.
3. " " 1218 hundredths, 1218 millionths.
4. " " 9 ten-thousandths, $\frac{9}{1000}$ and $\frac{19}{10}$.
5. " " 1019 tenths, 1019 hundredths, 1019 thousandths.

108. TO EXPRESS A DECIMAL AS A VULGAR FRACTION, find the name of the last figure used, and write $\frac{\text{number}}{\text{name}}$.

Express as vulgar fractions ·037, ·01, 2·3, 3·17.

$$\underset{th^{ths}}{\cdot037} = \frac{37}{1000}, \qquad \underset{h^{th}}{\cdot01} = \frac{1}{100},$$

$$\underset{t^{ths}}{2\cdot3} = \frac{23}{10} \text{ or } 2\frac{3}{10}, \qquad 3\cdot17 = \frac{317}{100} \text{ or } 3\,\frac{17}{100}.$$

Ex. LXIII. Express as vulgar fractions in their lowest terms. 91.

1. ·107, ·17, ·0017, ·01219.
2. 3·9, 4·175, 5·135, 5·117.
3. ·0001, ·017, ·000005, ·001.
4. 15·25, 18·2, 134·25, 53·155.
5. 8·125, 10·005, 15·05, 7·0015.
6. 10·1, 100·001, 7·105, 6·0055.

ADDITION AND SUBTRACTION.

109. Place the decimal points under one another in a vertical line, and then *like will be arranged under like.* Add and subtract as with integers. 10, 13.

$$\begin{array}{cccccccccccccc} & h & t & & h & t & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bullet & 0 & 0 & 0 & 0 & 0 & 0 \\ M & & & TH & & & U & & t^{ths} & h^{ths} & th^{ths} & t\text{-}th^{ths} & h\text{-}th^{ths} & m^{ths} \end{array}$$

$$\begin{array}{r} 3\cdot415 \\ \cdot931 \\ 7\cdot \\ 5\cdot125 \\ 3\cdot077 \\ \hline 18\cdot548 \end{array} \qquad \begin{array}{lll} 18 \text{ thous}^{dths} & \text{i.e.} & \underset{h^{dth}}{1}\ \ \underset{th^{dths}}{8.} \\[2ex] 14 \text{ hund}^{dths} & \text{i.e.} & \underset{t^{th}}{1}\ \ \underset{h^{dths}}{4.} \\[2ex] 15 \text{ tenths} & \text{i.e.} & \underset{u}{1} \bullet \underset{t^{ths}}{5.} \end{array}$$

Ex. LXIV. Find the sum of

1. 13·355 + ·5127 + ·00005 + 1 + ·0015 + 37·5.
2. 37·537 + 3·7537 + 375·37 + 3753·7 + 37537.
3. ·0001 + 1·011 + 111 + 1011· + ·111 + ·1011.
4. 1·01234 + 3·5123 + ·5755 + 13·575 + 13·5.
5. ·32174 + 3·2174 + 32·174 + ·00321 + ·03217.
6. 15·125 + 151·25 + 1512·5 + 15125 + ·00125.

$$\begin{array}{lcccccccc} \text{From} & 1 & 5 & 7 & \cdot & 2^{10} & 5 & 5^{10} & 0^{10} \\ \text{Take} & & 1 & 5^{1} & \cdot & 7 & 2^{1} & 5^{1} & 5 \\ \cline{2-9} & 1 & 4 & 1 & \cdot & 5 & 2 & 9 & 5 \end{array} \qquad \begin{array}{ll} 10 \text{ t-th}^{ths} & = 1\text{th}^{th} \\ 10 \text{ th}^{ths} & = 1\text{h}^{th} \\ 10 \text{ t}^{ths} & = 1 \text{ u.} \end{array}$$

Ex. LXV.

1. 15·002 − 11·777.
2. 2·25 − ·751325.
3. 171·1514 − ·17151.
4. 3·1425 − 2·889765.
5. 16·5543 − 15·4755.
6. 2158·6 − ·25168.
7. 1·31625 − 1·25613.
8. 2·005 − 1·999.
9. ·00005 − ·0000125.
10. 15·0165 − 3·00055.
11. 12·015 − 8·7557.
12. 3·121515 − 1·563755.

MULTIPLICATION.

110. Multiply as if all the figures were integers, and mark off in the product as many decimal figures as are found in the multiplicand and multiplier together.

Multiply 15·5 by 3·25.

$$\begin{array}{r} 15\cdot5 \\ 3\cdot25 \\ \hline 775 \\ 310 \\ 465 \\ \hline 50\cdot375 \end{array} \qquad \begin{array}{l} = \frac{155}{10} \\ = \frac{325}{100} \end{array} \ (108)$$

as vulgar fractions, $\frac{155}{10} \times \frac{325}{100} = \frac{50375}{1000}$ and 50375 thousandths will be written decimally by placing the last figure 5 in the thousandth's place, thus: 50·375, and the number of decimal figures will be found as many as in multiplicand and multiplier together.

Ex. LXVI.—Find the value of

1. 15·5 × 15·25.
2. 3·25 × 3·25
3. ·0001 × ·05.
4. ·00005 × ·005.
5. 22·25 × 23·5.
6. ·0125 × ·7.
7. 2·5 × 5·2 × ·05.
8. ·2 × ·04× ·05 × ·006.
9. 17·5 × 3·25 × 18.
10. ·005 × ·5 × ·015 × ·009.
11. 18·7 × 13·5 × ·55.
12. 1·15 × ·35 × ·5 × 1·5.

DIVISION.

111. Work as with integers, remembering that the *last figure* used *gives* the *name of* the particular *number used and* also *of* the *quotient figure.*

$$\begin{array}{cccccccccccccc} & \text{h} & \text{t} & & \text{h} & \text{t} & & & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \bullet & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{M} & & & \text{TH} & & & \text{U} & & \text{t}^{\text{ths}} & \text{h}^{\text{ths}} & \text{th}^{\text{ths}} & \text{t-th}^{\text{ths}} & \text{h-th}^{\text{ths}} & \text{m}^{\text{ths}} \end{array}$$

$$\begin{array}{r} 5\,)\,4\cdot235 \\ \hline \cdot847 \end{array} \quad \Bigg| \quad \begin{array}{l} 5\,)\,42 \text{ tenths} \\ \quad 8 \text{ tenths, 2 over} \end{array} \qquad \begin{array}{l} 5\,)\,23 \text{ hundredths} \\ \quad 4 \text{ hun}^{\text{dths}}\text{, 3 over} \end{array} \qquad \begin{array}{l} 5\,)\,35 \text{ thousandths} \\ \quad 7 \text{ thousandths} \end{array}$$

In all cases it is sufficient to work down to millionths, unless directed to do otherwise.

Ex. LXVII.

1. 131·255 ÷ 5.
2. 3120·005 ÷ 7.
3. 120·008 ÷ 8.
4. 476·136 ÷ 12.
5. 151·1232 ÷ 6.
6. 100·0005 ÷ 9.
7. 300·0123 ÷ 8.
8. ·412675 ÷ 9.
9. ·1304276 ÷ 12.

LONG DIVISION.

112. Observing that the quotient remains the same, when divisor and dividend are both multiplied by 10, if the divisor is not a whole number, *multiply it by* 10 *as many times as* may be *necessary to make* it a *whole number;* and *multiply* the *dividend* also *by* 10, the SAME *number of times.* The division can then be carried on as in integers; the **last figure used gives** the name of the number under division, and also the **name of the figure obtained as a quotient,** for a certain part of tenths, must be tenths, of hundredths, hundredths, &c.

2) 6 (3
10 10
— —
20) 60 (3
10 10
—— ——
200) 600 (3
&c.

113. The multiplication of the divisor by 10 once or more, as also of the dividend, can be done at once, by a change of the position of the figures, with reference to the points. 106.

Divide 4·785 by ·15.

·15) 4·785 (when multiplied by 10 × 10 become 15·) 478·5 (31·$\overset{t}{9}$
45
—
28 units
15
—
135 tenths
135
═

The division consists in finding
$\frac{1}{15}$th part of 47 tens = 3 tens
" " 28 units = 1 unit
" " 135 tenths = 9 tenths.

Ex. LXVIII. Find the value of

1. 31·250 ÷ 125.
2. ·15625 ÷ 12·5.
3. 156·25 ÷ ·025.
4. 1·5625 ÷ ·0025.
5. 181·3 ÷ ·185.
6. 362·6 ÷ ·00074.
7. 242·76 ÷ 2856.
8. 24·276 ÷ 2·856.
9. 48·552 ÷ 14·28.
10. 12·138 ÷ ·956.
11. ·375 ÷ ·000375.
12. 75000 ÷ ·75.

114. To REDUCE A FRACTION TO A DECIMAL, consider the fraction as $\frac{\text{dividend}}{\text{divisor}}$ (81*b*), and divide the numerator by the denominator (111) (113).

Reduce $\frac{3}{8}$ and $8\frac{5}{16}$ to a dec.

$$8)\underline{3{\cdot}000}$$
$${\cdot}375 \text{ being } \tfrac{1}{8}\text{th of 30 tenths}$$
$$\text{,, ,, 60 hundredths}$$
$$\text{,, ,, 40 thousandths.}$$
$$\tfrac{3}{8} = {\cdot}375$$

$$16 \left| \begin{array}{l} 4)\underline{\,5{\cdot}00} \\ 4)\underline{\,1{\cdot}2500} \\ \quad {\cdot}3125 \end{array} \right.$$
$$8\tfrac{5}{16} = 8{\cdot}3125.$$

Ex. LXIX. Reduce to decimals

1. $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{3}{20}$.
2. $\frac{103}{125}, 11\frac{12}{13}, \frac{3000}{25}, 3\frac{1}{16}$
3. $4\frac{11}{12}, \frac{113}{118}, \frac{1}{117}, 12\frac{21}{35}$.
4. $\frac{1}{213}, \frac{1}{1001}, \frac{13}{1003}, \frac{7}{1314}$
5. $13\frac{1}{7}, 12\frac{3}{8}, 10\frac{1}{20}, 5\frac{4}{5}$.
6. $3\frac{1}{28}, 4\frac{2}{175}, 5\frac{15}{16}, 8\frac{11}{120}$
7. $2\frac{1}{2}$ of $3\frac{1}{4}$ of $4\frac{1}{8}$.
8. $5\frac{1}{4}$ of $\frac{1}{25}$ of $\dfrac{2\frac{1}{2}}{3\frac{1}{2}}$.

CIRCULATING DECIMALS.

115. When, as in bringing $\frac{5}{12}$ to a dec., $\dfrac{12)\,5{\cdot}000}{{\cdot}41\dot{6}}$ a figure recurs, it is usual to write it only once, and to place a dot over it, to signify, that if the division were carried on, there would be a continued repetition of that figure; should two figures recur, a dot is placed over each; when three figures or more recur, a dot over the first and one over the last figure indicate the repetition.

116. To REDUCE A DECIMAL, IN WHICH ALL THE FIGURES CIRCULATE, TO A VULGAR FRACTION, write as a numerator the circulating figure or figures, and as a denominator 9, 99, 999, &c., just as one, two, three, &c., figures circulate.

The correctness of the method will appear from the following considerations:—

$$\tfrac{1}{9} = \cdot 1111111,\ \&c. \qquad \therefore\ \tfrac{5}{9} = \cdot 5555555,\ \&c.$$
$$\tfrac{1}{99} = \tfrac{1}{9} \div 11 = \cdot 010101,\ \&c. \qquad \therefore\ \tfrac{5}{99} = \cdot 050505,\ \&c.$$
$$\tfrac{1}{999} = \tfrac{1}{9} \div 111 = \cdot 001001,\ \&c. \qquad \therefore\ \tfrac{5}{999} = \cdot 005005,\ \&c.$$

that is, in the vulgar fractions, which are equivalent to circulating decimals, as $\frac{5}{9} = \cdot\dot{5}$, $\frac{5}{99} = \cdot\dot{0}\dot{5}$, the recurring figure is found in the numerator, and the number of 9s in the denom^r corresponds to the number of repeat^s figures.

Reduce to a vulgar fraction $\cdot\dot{2}$. $\dot{0}\dot{5}$, $\cdot\dot{0}24\dot{9}$.

$$\cdot\dot{2} = \tfrac{2}{9} \qquad \cdot\dot{0}\dot{5} = \frac{05}{99} \text{ i.e. } \frac{5}{99} \qquad \dot{0}24\dot{9} = \frac{0249}{9999} \text{ i.e. } \frac{249}{9999} = \frac{83}{3333}$$

Ex. LXX. Reduce to vulgar fractions

1. $\cdot\dot{3}\dot{7}$, $\cdot\dot{1}\dot{5}$, $\cdot\dot{1}1\dot{9}$, $3\cdot\dot{3}\dot{4}$, $5\cdot\dot{1}2\dot{7}$.
2. $4\cdot\dot{0}\dot{8}$, $5\cdot\dot{8}$, $13\cdot\dot{0}\dot{3}$, $7\cdot\dot{0}0\dot{8}$, $3\cdot\dot{0}\dot{4}$.
3. $14\cdot\dot{1}\dot{4}$, $7\cdot\dot{2}\dot{3}$, $\cdot\dot{1}1\dot{7}$, $4\cdot\dot{2}\dot{6}$, $19\cdot\dot{1}\dot{9}$.

117. TO REDUCE TO A VULGAR FRACTION A DECIMAL, OF WHICH SOME FIGURES CIRCULATE AND SOME DO NOT,—leaving integers untouched, *use* the *decimal figures* to the end of the first period only, that is, *without any repetition of figures*—for the Numerator, write down all the decimal figures and subtract from them the non-repeating figures; for the Denominator, write of 9s one for each repeating figure, followed by 0s, one for each non-repeating figure.

$3\cdot\dot{4} = 3\tfrac{4}{9}$. (116).
$3\cdot 0\dot{4} = 3$ and $(\tfrac{4}{9} \div 10) = 3\tfrac{4}{90}$.
$3\cdot 00\dot{4} = 3$ and $(\tfrac{4}{9} \div 100) = 3\tfrac{4}{900}$

In these denom$^{rs.}$ are found one 9 for the one repeating figure, and cyphers for each non-repeating figure.

$$\cdot 5\dot{4}\dot{3} = \frac{5}{10} + \frac{43}{990} = \frac{4950 + 430}{9900} = \frac{5380}{9900} = \frac{538}{990} \text{ i.e. } \frac{543 - 5}{990}$$

$$\text{now } \frac{543-5}{990} = \frac{\textit{all dec}^{d}\textit{ figures} - \textit{non-repeat}^{g}\textit{ figures,}}{\textit{9s for each repeat}^{g}\textit{, and 0s. for each non-repeat}^{g}\textit{ figure.}}$$

Ex. LXXI. Reduce to vulgar fractions in their lowest terms.

1. $\cdot 34\dot{1}$, $3\cdot 4\dot{1}\dot{6}$, $1\cdot 1\dot{5}\dot{5}$, $4\cdot 03\dot{5}$.
2. $2\cdot 73\dot{5}$, $5\cdot 2\dot{6}\dot{5}$, $15\cdot 12\dot{9}$, $3\cdot 2\dot{5}$.
3. $1\cdot 01\dot{2}\dot{5}$, $7\cdot 3\dot{1}5\dot{3}$, $8\cdot 13\dot{1}2\dot{3}$, $17\cdot 11\dot{6}$.
4. $3\cdot 218\dot{4}$, $3\cdot 2\dot{1}8\dot{4}$, $3\cdot\dot{2}18\dot{4}$, $3\cdot 21\dot{8}\dot{4}$.
5. $\cdot 042\dot{3}$, $\cdot 04\dot{2}\dot{3}$, $\cdot 0\dot{4}2\dot{3}$, $4\cdot 04\dot{2}$.
6. $4\cdot 11\dot{5}$, $7\cdot 2\dot{1}\dot{5}$, $6\cdot 15\dot{1}\dot{5}$, $15\cdot 3\dot{3}$.

118. In Examples of add^n and subt^n, in which circulating decimals occur, and the result is required correct to six places, it will be sufficient to work as far as 8 or 9 places.

Add together $14\cdot\dot{2}$, $3\cdot 0\dot{1}\dot{7}$, $112\cdot 000\dot{3}$, $2\cdot\dot{5}1\dot{8}$, correctly to six places. From $1\cdot 15\dot{1}2\dot{3}$ take $\cdot\dot{5}2\dot{7}$.

$$\begin{array}{r|l} 14\cdot 222222 & 22 \\ 3\cdot 017171 & 71 \\ 112\cdot 000333 & 33 \\ 2\cdot 518518 & 51 \\ \hline 131\cdot 758245 & 77 \end{array} \qquad\qquad \begin{array}{r|l} 1\cdot 151231 & 23 \\ \cdot 527527 & 52 \\ \hline \cdot 623703 & 71 \end{array}$$

Ex. LXXII. Find the value correct to six places, of

1. $\cdot 15\dot{7} + \cdot 1325\dot{1}\dot{4} + 3\cdot 51\dot{7} + 24\cdot\dot{3}\dot{6} + \cdot 00151\dot{2}7\dot{6}$.
2. $36\cdot 2\dot{5} + \cdot 2\dot{7} + 7\cdot 15\dot{3} + \cdot\dot{8} + 3\cdot 7\dot{5} + 142\cdot\dot{1}\dot{6}$.
3. $\cdot\dot{3} - \cdot 00\dot{5}$; $\cdot\dot{0}\dot{7} - \cdot 01\dot{2}34\dot{5}$; $9 - 6\cdot 1\dot{2}5\dot{7}$; $\cdot\dot{7} - \cdot 01\dot{7}$.

119. When *absolute correctness* is *required, and in* examples of *multiplication* and *division*, it is necessary to change the circulating decimals to vulgar fractions (116, 117), and to work with them, changing the result back to a decimal, if required to do so.

Find the value of $\cdot 3\dot{6} \times \cdot\dot{3}\dot{5}$ and of $\cdot 1\dot{6} \div \cdot 00\dot{3}$.

$$\cdot 3\dot{6} \times \cdot\dot{3}\dot{5} = \frac{\overset{1}{\cancel{33}}}{\underset{18}{\cancel{90}}} \times \frac{\overset{7}{\cancel{35}}}{\underset{3}{\cancel{99}}} = \frac{7}{54} = \cdot 129629, \text{\&c.}$$

$$\cdot 1\dot{6} \div \cdot 00\dot{3} = \frac{15}{90} \div \frac{3}{900} = \frac{\overset{5}{\cancel{15}}}{\underset{1}{\cancel{90}}} \times \frac{\overset{10}{\cancel{900}}}{\underset{1}{\cancel{3}}} = 50.$$

Ex. LXXIII. Find the value of

1. $36 \cdot 2\dot{5} \times \cdot\dot{3}$; $4 \cdot \dot{7}\dot{5} \times \cdot 2\dot{3}$; $15 \cdot \dot{1}\dot{5} \times \cdot 3\dot{1}$.
2. $3 \cdot 12\dot{9} \times \cdot 3\dot{8}$; $71 \cdot 2\dot{7} \times \cdot \dot{1}\dot{8}$; $172 \cdot \dot{1} \div \cdot 1\dot{6}$.
3. $\cdot\dot{3} \div \cdot\dot{0}\dot{9}$; $\cdot 01\dot{6} \div 2 \cdot \dot{6}$; $3 \cdot 1\dot{7} \div 4 \cdot \dot{1}\dot{8}$.
4. $6 \cdot 1\dot{2}\dot{6} \div 3 \cdot 0\dot{4}$; $4 \cdot \dot{1} \div 1 \cdot \dot{4}$; $18 \cdot 1\dot{7}\dot{5} \div 18 \cdot \dot{1}$.

120. TO FIND THE VALUE OF ANY DECIMAL OF A GIVEN QUANTITY, work as with integers, in Reduction for, by way of illustration, the change from a part of a £ to shillings, must be made by the same method, as from whole £ to shillings.

Find the value of £·3675.

$$\begin{array}{rll}
 & \text{£} & \\
 & \cdot 3675 & \text{or part of a £.} \\
 & 20\cdot & \\
\hline
s. & 7 \cdot 35\cancel{0}\cancel{0} & \text{or } 7s. \text{ and part of another shilling.} \\
 & 12\cdot & \\
\hline
d. & 4 \cdot 2\cancel{0} & \text{or } 4d. \text{ and part of another penny.}
\end{array}$$

$$\cdot 2 = \frac{\text{number}}{\text{name}} = \tfrac{2}{10} = \tfrac{1}{5}. \qquad 7s.\ 4\tfrac{1}{5}d.$$

121. Whenever *cyphers* in decimal places are not of use to keep figures in their proper positions with reference to the decimal points they should be ***struck out*** as above, because *of no value and of no use.*

Find the value of ·05 of £2 5s. and ·1255 of 7s. 6d.

(*a.*) Quantities &c. must be expressed in ONE NAME for *multiplication and division by decimals.*

$$\begin{array}{rl}
 & s. \\
 & 45 \\
 & \cdot 05 \\
\hline
s. & 2 \cdot 25 \\
 & 12\cdot \\
\hline
d. & 3 \cdot 00
\end{array}
\qquad\qquad
\begin{array}{rl}
 & d. \\
 & 90\cdot \\
 & \cdot 1255 \\
\hline
d. & 11 \cdot 295\cancel{0}
\end{array}$$

$$\cdot 295 = \tfrac{295}{1000} = \tfrac{59}{200}.$$

Find the value of $2 \cdot 3\dot{8}$ of a ton.

$$2 \cdot 3\dot{8} = 2\tfrac{35}{90} = 2\tfrac{7}{18} \text{ of a ton.} \quad 103b.$$

$$\begin{array}{r|l|rrrrrr}
 & & \text{tons} & \text{cwt.} & \text{qrs.} & \text{lbs.} & \text{oz.} & \text{drs.} \\
18 & 3) & 7 & 0 & 0 & 0 & 0 & 0 \\
 & 6) & 2 & 6 & 2 & 18 & 10 & 10\frac{2}{3} \\
\hline
 & & & 7 & 3 & 3 & 1 & 12\frac{4}{9} \\
 & & 2 & 0 & 0 & 0 & 0 & 0 \\
\hline
 & & 2 & 7 & 3 & 3 & 1 & 12\frac{4}{9} \\
\hline
\end{array}$$

$$2\tfrac{2}{3} = \tfrac{8}{3} \div 6 = \frac{\overset{4}{\cancel{8}}}{3} \times \frac{1}{\underset{3}{\cancel{6}}} = \frac{4}{9}.$$

Ex. LXXIV. Find the value of

1. ·35 of £1; ·1215 of £1.
2. 23·5 of 7*s.*; 1·75 of 3*s.* 6*d.*
3. ·1235 of £7; ·0075 of 9 guineas.
4. 2·35 of 18*s.* 6*d.*; 5·175 of 5½ guineas.
5. 18·125 of a cwt.; ·425 of 7½ tons.
6. 3·225 of £4 2*s.* 6*d.*; 23·25 of £8 3*s.* 6*d.*
7. ·175 of £6 6*s.* 8½*d.*; 4·4̇ of £2 3*s.* 6*d.*
8. 13·1255 of 10*s.* 6*d.*; ·00123125 of £10 6*s.* 8*d.*
9. ·75 of 1 m. 430 yds.; 5·5̇ of 1 cwt. 1 qr. 4 lbs.
10. ·1775 of 1 sq. yd.; 32·55 of 2 m. 0 fur. 10 p.
11. ·2275 of £12 17*s.* 6d.; 5·75 times £2 16*s.* 8*d.*
12. 37·45 of 10½*s.*; ·7*s.* + £·7 + ·7 crowns.
13. £·125 + ·125*s.* + ·125*d.*; ·125̇ of a guinea.
14. 2·1375 of 3*s.* 4*d.* + 5·385 of 6*s.* 8*d.* − ·325 of £1.
15. ·275 of a guinea + 1·275 of a £ + 1·275 of a crown.

122. TO REDUCE A GIVEN QUANTITY TO THE DECIMAL OF ANOTHER QUANTITY, in some cases, the result can be obtained by a number of direct changes (Reduction, 38) from the lowest name in the given quantity to the name required, integers being added to decimals (like to like) as opportunities occur,—otherwise the given quantity must be expressed as a vulgar fraction of the other (102), and the vulgar fraction changed to a decimal (114).

(*a*) Reduce £3 15*s.* 6¾*d.* to the dec. of £1.

4) 3·	fourths of 1*d.*
12) 6·75	·75*d.* with 6·*d.* added.
20) 15·5625	·5625*s.* „ 15·*s.* „
3·778125	·778125£ with £3· added.

(*b*) Reduce 1*s.* 9¾*d.* to the dec. of 3*s.* 6½*d.*

1*s.* 9¾*d.* = 87 far.
3*s.* 6½*d.* = 170 far.

(By 102) vulg. fr. = $\frac{87}{170}$.

```
170·) 87·00 (·51, &c.
      850
      ———
       200
       170
       ———
        30
```

Ex. LXXV. Reduce—

1. 4*s*. 6*d*. to dec. of £1; £1 3*s*. 6*d*. to dec. of £1.
2. 18s. to dec. of 13*s*. 6*d*.; 14*s*. 6$\frac{1}{2}$*d*. to dec. of £1.
3. £1 3*s*. 6*d*. to dec. of £1; 3*s*. 6$\frac{3}{4}$*d*. to dec. of 15*s*. 9*d*.
4. 4*s*. 9$\frac{3}{4}$*d*. to dec. of 5*s*. 11$\frac{1}{2}$*d*.; £3 16*s*. 8*d*. to dec. of £5 19*s*. 8*d*.
5. 17*s*. 6$\frac{1}{2}$*d*. to dec. of 1 guinea; 7*s*. 6$\frac{3}{4}$*d*. to dec. of £4.
6. 5 c. 1 qr. to dec. of 13 c. 2 qrs.; 2 fur. 39 yds. to dec. of 1 mile.
7. 4 m. 1517 yds. to dec. of 100 m.; 8 hrs. 18 m. 16 sec. to dec. of 1 day.
8. 2 qrs. 1 lb. to dec. of 1 cwt.; 15$\frac{1}{2}$ days to dec. of 365 days.
9. 13*s*. 8$\frac{1}{2}$*d*. to dec. of £1; 16*s*. 9$\frac{3}{4}$*d*. to dec. of 4 guineas.
10. 2$\frac{1}{2}$ guin. to dec. of 5$\frac{1}{4}$ guin.; 3 lbs. 7 oz. to dec. of 5$\frac{1}{2}$ cwt.
11. 18*s*. 6*d*. to dec. of 1 crown; 15*s*. 4*d*. to dec. of $\frac{1}{2}$ guin.
12. 2$\frac{1}{4}$ in. to dec. of 10 yds; 2 t. 1 c. 8 lb. to dec. of 1$\frac{1}{2}$ cwt.

MISCELLANEOUS EXAMPLES (LXI—LXXV).

1. Write as decimals $\frac{17}{100}$, $\frac{117}{10}$, $\frac{3}{10000}$, and 13$\frac{1}{1000}$. 107.
2. Express as decimals $\frac{3}{8}$, $\frac{4}{25}$, $\frac{1}{50}$, $\frac{1}{125}$. 114.
3. Express as a decimal 3$\frac{1}{2}$ of 4$\frac{1}{3}$ of 5$\frac{1}{8}$ of 3. 89, 114.
4. Express as a decimal $\frac{4\frac{1}{5}}{3\frac{1}{8}}$ of 7$\frac{3}{4}$ of 5$\frac{1}{2}$. 81, 89, 114.
5. Find the sum of 121053, 1·21053, 12·1053, 1210·53. 109.
6. Find the continued product of 12·5, 1·25, and 125. 110.
7. Divide ·0125 by ·155. 112, 113.
8. Divide 178·25 by 41·5. 112, 113.
9. Express 3·125 and $3\cdot12\dot{5}$ as vulgar fractions. 108, 117.
10. Find the value of £·31275. 120.
11. Reduce $\frac{27}{28}$ and $\frac{28}{27}$ to decimals. 114.
12. Find the value of £·05 + ·125*s*. + 3·25*d*. 120.
13. Multiply ·25 by ·35, and divide the product by ·075. 110, 113.
14. Find the value of £3125 + ·3125*s*. + 31·25*d*. 120.
15. Reduce 4*s*. 6$\frac{1}{2}$*d*. to the decimal of a guinea. 122.
16. Find the number of hours, minutes, and seconds equal to ·1715 days. 120.
17. Multiply 1*s*. 6$\frac{1}{2}$*d*. by ·125, and divide £8 15*s*. 6*d*. by ·75. 121 Ex.
18. Find the value of 1·25 of 3·75 of £1·5*s*. 110, 120.
19. Find the value of $1\cdot2\dot{7} \times 3\cdot\dot{3}$ of £4$\frac{1}{2}$. 117, 116, 98.
20. Find the value of ·73125 of £15. 121.
21. Find the difference between 1·5 of 3·5 of £1·7*s*., and $\frac{1}{9}$ of 3$\frac{6}{7}$ of 2$\frac{1}{3}$ of £18. 110, 120, 98, 103.
22. Multiply $8\cdot\dot{8}$ by 12·5 (116, 108), and divide the result by $\frac{7\cdot5}{\cdot25}$. (81 *b*).

23. Find the value of ·475 of £5·355 (121), and of ·12345 of £100 (121), and reduce 1·5*d.* to the decimal of £3·75. 122.

24. Reduce $\frac{3}{25}$ and $\frac{7}{20}$ to decimals, also 4·17 and $4{\cdot}1\dot{7}$ to vulgar fractions. 114, 108, 117.

25. Express ·125 of a ton in half ounces, and find their value at 13·5*d.* per half oz. 120, 110.

26. The length of a floor being 17·75 feet, and the width 12·125 feet, find the number of square feet of surface. 65, 110, 120.

27. Multiply 5*s.* 0·15*d.* by 75·75, and divide £13 15$\frac{3}{4}$*s.* by ·005 of 1·5. 121*a*, 110, 112.

28. Add together $\frac{2}{3}$, $\frac{3}{8}$, $\frac{9}{10}$, $\frac{7}{100}$ as vulgar fractions and as decimals. 94, 114, 109.

29. Express ·065 cwt. as lbs., and multiply £1 5*s.* 7·75*d.* by 3$\frac{1}{4}$. 120, 108, 103*d.*

30. Reduce ·65 and ·0065 to vulgar fractions, and express £1 3*s.* 7$\frac{1}{2}$*d.* as a decimal of £4. 108, 122.

31. Find the value of £·015 + ·015*s.* + ·015 of a guinea. 120.

32. Reduce £·5 of £3 7*s.* 6*d.* to the decimal of ·75 of £5 6*s.* 121*a*, 122.

33. The multiplicand is ·000025 and the product ·0075; find the multiplier. 33*c.*

34. Multiply 1 cwt. 1 qr. 4 lb. by 3·125, and divide 4·17235 by ·0005. 121*a*, 112.

35. Reduce $2{\cdot}\dot{5}$ of £·85 to the decimal of $1{\cdot}\dot{5}$ of £1·5. 116, 108, 102.

36. Reduce $\cdot\dot{3}$ of 1*s.* + $\cdot1\dot{6}$ of half-a-crown + ·5*d.* to the decimal of £10. 116, 117, 108, 102, 114.

37. Find the value of $\frac{1\cdot75}{1\cdot25}$ of $\frac{1\cdot5}{\cdot75}$ of $\frac{4\cdot5}{\cdot15}$. (81 *b*), 112, 110.

38. Reduce to a simple quantity $\frac{3\cdot375}{2\cdot5}$ of $\frac{4\cdot5}{\cdot15}+\frac{3\cdot8}{\cdot2}$. (81*b*), 112, 110, 112.

39. Find the difference between 5$\frac{1}{2}$ guin. and £3·15. 103*b*, 120.

40. Express £17 8*s.* 10$\frac{1}{2}$*d.* as a decimal of £23 15*s.* 7$\frac{1}{2}$*d.* 122*b*.

41. Express as vulgar fractions $\cdot\dot{1}2\dot{7}$, $\cdot\dot{3}3\dot{9}$, $18{\cdot}\dot{1}21\dot{5}$. 116.

42. Express as vulgar fractions $3{\cdot}01\dot{7}$, $15{\cdot}012\dot{3}$. 117.

43. Divide the sum of 18·15 and 1·815 by their difference.

44. By how much is the sum of 1$\frac{1}{8}$, 3·25, 4·125, greater than 2·1325? 114, 109.

45. Work correctly to six places of decimals $3{\cdot}12\dot{1}\dot{5}$ – $2{\cdot}1\dot{2}1\dot{5}$ + $14{\cdot}\dot{5}$. 118.

46. Find the value of £1·75 + 2·1625 guin. – 1·215 crowns. 120.

47. Reduce ·00315 cwt. to oz. and decimal of an oz. 120.

48. Express 17·125 miles in yards. 120.

49. Reduce 3·125 guin. to £ and decimal of a £. 48, 120.

50. Express $£3{\cdot}7\dot{3}$ as a decimal of $3{\cdot}7\dot{3}$ guineas. 117, 102*c*, 114.

PRACTICE.

123. Practice is the name given to the method of finding the value of any quantity, *when the price given is the value of a unit*, as the value of 37 c. 3 qrs. 7 lbs. at £1 1*s*. 8*d*. for 1 *cwt*., or of 253 yds, at 4*s*. 6½*d*. for 1 *yd*.

Taking the value of a unit as material to work with, the result required is reasoned out with such considerations as the following:—

to three times the quantity	belongs	three times the value.	
four times, &c. „	„	four times, &c.	„
one-third „	„	one-third	„
one-fourth, &c. „	„	one-fourth, &c.	„
three times, &c. the price	„	three times, &c.	„
one-third, &c. „	„	one-third, &c.	„

of £1 i.e. 20*s*.		
s.	*d*.	
10	0	= $\frac{1}{2}$
6	8	= $\frac{1}{3}$
5	0	= $\frac{1}{4}$
4	0	= $\frac{1}{5}$
3	4	= $\frac{1}{6}$
2	6	= $\frac{1}{8}$
2	0	= $\frac{1}{10}$
1	8	= $\frac{1}{12}$
1	4	= $\frac{1}{15}$
1	3	= $\frac{1}{16}$
1	0	= $\frac{1}{20}$
	8	= $\frac{1}{30}$
	6	= $\frac{1}{40}$

of 1*s*. i.e. 12*d*.	
d.	
6	= $\frac{1}{2}$
4	= $\frac{1}{3}$
3	= $\frac{1}{4}$
2	= $\frac{1}{6}$
1½	= $\frac{1}{8}$
1	= $\frac{1}{12}$

of 1 ton, i.e. 20 cwt.	
cwt.	
10	= $\frac{1}{2}$
5	= $\frac{1}{4}$
4	= $\frac{1}{5}$
2½	= $\frac{1}{8}$
2	= $\frac{1}{10}$
1	= $\frac{1}{20}$

of 1 qr. i.e. 28 lbs.	
lbs.	
14	= $\frac{1}{2}$
7	= $\frac{1}{4}$
4	= $\frac{1}{7}$
3½	= $\frac{1}{8}$
2	= $\frac{1}{14}$
1	= $\frac{1}{28}$

of 1 cwt. i.e. 112 lbs.	
2 qrs. (56 lbs.)	= $\frac{1}{2}$
1 qr. (28 „)	= $\frac{1}{4}$
16 „	= $\frac{1}{7}$
14 „	= $\frac{1}{8}$
8 „	= $\frac{1}{14}$
7 „	= $\frac{1}{16}$
4 „	= $\frac{1}{28}$

124. In all cases exact parts may be found by expressing both quantities in the same name, and trying consecutive numbers; thus: having 18 *sec*. to get an exact part of 1 minute, *i.e.* 60 *sec*., try 18, 17, 16, and then 15 are found exactly ¼th.

Find the value of 45 yards at £3 15s. per yd.

	£		
10s. = ½ of £ .	45		value of 45 yds. at £1 each.
	3		
	135		„ „ £3 each.
5s. = ½ of 10s.	22	10	„ „ 10s. „
	11	5	„ „ 5s. „
	168	15	£3 15s. each.

Considerations that occur in example above, starting with value at £1 each (worked out mentally).

Value at £3 each = three times value at £1 each.
„ 10s. „ = one-half „ £1 „
„ 5s. „ = one-half „ 10s. „

Ex. LXXVI.

1. 138 @ £5 10s.
2. 246 @ £3 6s. 8d.
3. 159 „ £2 2s. 6.
4. 175 „ £5 3s. 4d.
5. 873 „ £4 1s. 8d.
6. 258 „ £5 4s.
7. 140 „ £2 15s.
8. 347 „ £2 16s. 8d.
9 418 „ £1 7s. 6d.
10. 258 „ £3 12s.
11. 250 „ £7 1s. 8d.
12. 384 „ £4 16d.

Find the value of 1278 articles at £3 15s. 8½d. each.

10s. = ½ of £1	£1278 =			value of 1278 articles at £1 each.
	3			
	3834			„ £3 each.
5s. = ½ of 10s.	639			„ 10s.
6d. = ⅒ of 5s.	319	10		„ 5s.
2d. = ⅓ of 6d.	31	19		„ 6d.
½d. = ¼ of 2d.	10	13		„ 2d.
	2	13	3	„ ½d.
	4837	15	3	£3 15 8½

Considerations that occur in the foregoing example :

Value at £3 each = three times value at £1 each.
„ 10s. „ = one-half „ £1 „
„ 5s. „ = one-half „ 10s. „
„ 6d. „ = one-tenth „ 5s. „
„ 2d. „ = one-third „ 6d. „
„ ½d. „ = one-fourth „ 2d. „

Ex. LXXVII.

1. 225 at £5 12*s*. 6*d*.
2. 318 at £3 14*s*. 6*d*.
3. 146 at £1 15*s*. 6*d*.
4. 552 at £2 11*s*. 6*d*.
5. 358 at £6 17*s*.
6. 219 at £1 17*s*. 6*d*.
7. 356 at £9 14*s*. 6*d*.
8. 357 at £4 15*s*. 8*d*.
9. 257 at £1 5*s*. 6*d*.
10. 388 at £9 13*s*. 6*d*.
11. 528 at £5 17*s*. 9*d*.
12. 188 at £1 16*s*. 8*d*.

125. It is often desirable to consider whether the required result will be obtained most readily by starting with the value at £1 each, at 1*s*. each, or at 1*d*. each. The following example is worked out from the three different starting-points for the purpose of comparison,

Find the value of 77 yds. at 6*s*. 10½*d*. each.

	£
5*s*. ¼ of £1	77 at £1 each.
1*s*. ⅕ of 5*s*.	19 5
6*d*. ½ of 1*s*.	3 17
4*d*. ⅓ of 1*s*.	1 18 6
½*d*. ⅛ of 4*d*.	1 5 8
	3 2½
	26 9 4½

	s.
6*d*. ½ of 1*s*.	77 at 1*s*. each.
4*d*. ⅓ of 1*s*.	6
	462 at 6*s*. ,,
	38 6
½*d*. ⅛ of 4*d*.	25 8
	3 2½
2,0)	52,9 4½
	26 9 4½

	d.
½*d*. ½ of 1*d*.	77 at 1*d*. each.
	82
	154
	616
	6314 at 82*d*. each.
	38½ at ½*d*. ,,
12)	6352½
2,0)	52,9 4½
	26 9 4½

Ex. LXXVIII.

1. 337 at 3*s*. 1½*d*.
2. 328 at 14*s*. 6½*d*.
3. 536 at 3*s*. 9¾*d*.
4. 653 at 5*s*. 2½*d*.
5. 229 at 9*s*. 2½*d*.
6. 118 at 7*s*. 10½*d*.
7. 300 at 11*s*. 7¾*d*.
8. 1000 at 13*s*. 8¾*d*.
9. 1500 at 12*s*. 8½*d*.
10. 1509 at 18*s*. 10½*d*.
11. 788 at 8*s*. 8¾*d*.
12. 488 at 6*s*. 6¾*d*.

Ex. LXXIX.

1.	158 at £2 16*s*. 6*d*.	2.	316 at £3 16*s*. 6*d*.
3.	316 at £4 18*s*. 10*d*.	4.	528 at £7 14*s*. 8*d*.
5.	228 at £1 17*s*. 10½*d*.	6.	336 at £2 11*s*. 9½*d*.
7.	1329 at £5 9*s*. 8¼*d*.	8.	258 at £1 2*s*. 2½*d*.
9.	365 at £2 4*s*. 5½*d*.	10.	336 at £1 4*s*. 8¾*d*.
11.	338 at £5 6*s*. 6¾*d*.	12.	500 at £2 13*s*. 11½*d*.

Find the value of 3 cwt. 1 qr. 14 lbs. at £1 5*s*. 8*d*. per cwt.

	£	*s*.	*d*.	
1 qr. = ¼ of 1 cwt.	1	5	8	val. of 1 cwt.
			3	
14 lbs. = ½ of 1 qr.	3	17	0	val. of 3 cwt.
		6	5	„ 1 qr.
		3	2½	„ 14 lbs.
	£4	6	7½	„ 3 c. 1 qr. 14 lbs.

Considerations that occur.

3 times a quantity . .	3 times its value.
¼th of „ „ . .	¼th of „ „
½ of „ „ . .	½ of „ „

Find the rent of 15 ac. 3 r. 15½ p. at £1 15*s*. 4*d*. per acre.

	£	*s*.	*d*.	
2 r. = ½ of 1 ac.	1	15	4	rent of 1 ac.
			15	
	26	10	0	. . . 15 ac.
1 r. = ½ of 2 r.		17	8	. . . . . . 2 r.
10 p. = ¼ of 1 r.		8	10	. . . . . . 1 r.
5 p. = ½ of 10 p.		2	2½ = $\frac{20}{40}$	. . . . 10 p.
½ p. = $\frac{1}{10}$ of 5 p.		1	1¼ = $\frac{10}{40}$	. . . . 5 p.
			$1\frac{13}{40} = \frac{13}{40}$	. . . . . ½ p.
	27	19	$11\frac{3}{40}$	15 a. 3r. 15½ p.

Considerations by which the result is reached.

15 times a quantity of land	15 times its rent.
One-half of „ „	One-half of „
„ „ „ „	„ „ „
One-fourth of „ „	One-fourth of „
One-tenth of „ „	One-tenth of „

Ex. LXXX.

1. 5 cwt. 1 qr. 11 lbs. at £3 16*s*. per cwt.
2. 2 cwt. 3 qrs. 17 lbs. at £5 17*s*. 6*d*. per cwt.
3. 4 cwt. 2 qrs. 16 lbs. at £3 10*s*. 1½*d*. per cwt.
4. 12 cwt. 1 qr. 9 lbs. at £1 14*s*. 7¾*d*. per cwt.
5. 3 qrs. 4 lbs. 14 oz. at 17*s*. 8½*d*. per qr.
6. 2 qrs. 21 lbs. 8 oz. at £1 5*s*. 6½*d*. per gr.

7. 3 cwt. 1 qr. 7 lbs. 8 oz. at £1 14*s.* 8½*d.* per. qr.
8. 2 cwt. 1 qr. 4 lb. 5 oz. 3 drs. at £1 3*s.* 9*d.* per gr.
9. 12 oz. 16 dwts. 18 grs. at 13*s.* 9*d.* per oz.
10. 15 yds. 2 ft. 2 in. at 17*s.* 6*d.* per yd.
11. 47 a. 1 r. 28 p. at £3 5*s.* 6*d.* per acre.
12. 4 m. 3 w. 4 d. at £1 10*s.* 6*d.* per month.

MISCELLANEOUS EXAMPLES (LXXVI—LXXX).

1. What is the pay of 576 sailors, at 13*s.* 6½*d.* each?

2. Find the charge for 1718 barrels at 8*s.* 9¾*d.* each.

3. What will be the cost for 365 days at 5*s.* 8½*d.* per day?

4. A man owes £5000, and for every £ will pay only 17*s.* 6*d.*, what sum will pass to his creditors?

5. A man owes £1560, and will fail to pay 4*s.* 6*d.* on every £, how much in all will his creditors lose?

6. What will be the cost of papering 3125 sq. yds. at 6¾*d.* per yard?

7. What will be the value of 1255 cub. ft. of timber at 4*s.* 6½*d.* per cub. ft.?

8. Find 1237 times £1 16*s.* 8½*d.*

9. If one piece of gold coin weigh 3 dwts. 11 grs., what will be the weight of 576 pieces?

10. If lodgings cost 10*s.* 6*d.* per week, how much do they cost for 53 weeks 6 days?

11. If an officer's pay is 10*s.* 9*d.* per day, how much will be received in the year 1884?

12. If 3 cwt. 1 qr. 5 lbs. be bought for £1 1*s.* 6*d.* per cwt., and the same quantity at £2 3*s.* 9*d.* per cwt., what is the cost of the whole?

13. A room is 14 ft. sq. and 10 ft. high, find how many square yards of wall there are, and the expense of painting at 1*s.* 6¾*d.* per sq. yd. XLI. *x.*

14. 1000 acres are rented at £2 5*s.* 4*d.* per acre; and of them 803 are sub-let at £2 10*s.* 8*d.* per acre, and the rest at £2 12*s.* 6*d.* per acre, what gain is made?

15. A block of stone measures 14 ft. 9 in., 8 ft. 6 in., and 3 ft. 10., what will be its weight at 3 qrs. 7 lbs. per cub. ft.? 68*a.*

16. Find the value of 18 lbs. at 7½*d.* per lb., 28 lbs. at 3¼*d.*, 120 lbs. at 1*s.* 1½*d.*, and 15 lbs. 4 oz. at 4*s.* 7½*d.*

17. A chest of tea contains 139 lbs., find its value at 2*s.* 4½*d.* per lb.

18. What will be the cost of 1547 ac. 1 r. 18 p. at £3 15*s.* per ac.

19. What will be the cost of 9 cwt. 2 qrs. 17 lbs. of lead at £2 0*s.* 7*d.* per cwt.?

20. If a person's estate bring him in £21506 per annum; and the land tax is at the rate of 2*s.* 7½*d.* per £, how much will his tax amount to?

PROPORTION.

126. When two abstract or concrete numbers are compared as to the number of times the first contains the second, or as to the part the first is of the second, the result, obtained by division, is called the Ratio.

Thus such a *comparison* of 12 with 3 (written 12 : 3), or of 3 with 12 (written 3 : 12), *made by division*, gives the ratio of 12 : 3 as 4, that is, 12 is 4 times 3; and gives the ratio of 3 : 12 as $\frac{3}{12} = \frac{1}{4}$, that is, 3 is one-fourth of 12.

127. When two ratios differently expressed are equal, they form a Proportion, and the numbers, compared in them, are said to be Proportionals; thus 6 : 12 : : 13 : 26 are proportionals, since the ratios $\frac{6}{12}$ $\frac{13}{26}$ are equal, each being $\frac{1}{2}$.

128. When four numbers are proportionals, the product of the extremes is equal to the product of the means, thus:

$$6 : 12 :: 13 : 26 \qquad \begin{array}{r} \text{extremes } 6 \times 26 = 156. \\ \text{means } 12 \times 13 = 156. \end{array}$$

This will be seen by bringing the fractions which express the two ratios to a common denominator.

$$\begin{array}{rcc} & \frac{6}{12} & \frac{13}{26} \\ & \text{Com. Mult. } 12 \times 26 & \\ \text{extremes} & \frac{6 \times 26}{12 \times 26} & \frac{13 \times 12}{26 \times 12} \text{ means} \\ & \frac{156}{312} & \frac{156}{312} \end{array}$$

The fractions being equal, must have the *same numerators* when expressed with their *greatest* common denominator: and it will occur, in such work, that one numerator is the product of the two extremes; the other the product of the two means.

129. From this it follows that, of four numbers that are proportionals, any three being given, the fourth can be found by multiplication and division.

$$\overbrace{5 : \underbrace{15 :: 3} : 9} \qquad \text{extremes } 5 \times 9 = \text{means } 15 \times 3.$$

$$\frac{15 \times 3}{9} = \text{1st term.} \qquad {}^*(b)\ \frac{3 \times 15}{5} = \text{4th term.}$$

$$c)\ \frac{5 \times 9}{3} = \text{2nd term.} \qquad (d)\ \frac{5 \times 9}{15} \times \text{3rd term.}$$

* Used in working Rule of Three

Ex. LXXXI. Find the number required to complete the four proportionals in the following:

1. $65 : 260 :: 27 : x$.
2. $13 : 52 :: x : 144$.
3. $x : 27 :: 144 : 432$.
4. $9 : x :: 120 : 360$.
5. $55 : 65 :: 80 : x$.
6. $12 : 144 :: x : 1728$.
7. $15 : x :: 111 : 333$.
8. $x : 135 :: 2 : 18$.

130. To express the ratio of two concrete (36) quantities, such quantities must of necessity be of the *same kind* and in the *same denomination*, otherwise no exact comparison can be made between them.

The RATIO, giving the result of the comparison obtained by the division of the former term by the latter, IS AN ABSTRACT NUMBER being either a *certain number of times* or a *certain part.*

Find the ratio of 6*s.* 8*d.* : 13*s.* 4*d.*

6*s.* 8*d.* = 80*d.*
13*s.* 4*d.* = 160*d.*

$$\frac{80}{160} = \frac{1}{2} \text{ i.e. } 1 : 2.$$

Ex. LXXXII. Find the ratio of

1. 2*s.* 6*d.* : 5*s.*
2. £2 6*s.* 8*d.* : £3 13*s.* 4*d.*
3. 1 cwt. 1 qr. 8 lbs. : 2 cwt. 2 qrs. 16 lbs.
4. 3 qrs. 14 lbs. : 8 cwt.
5. 1 h. 45 m. 45 sec. : 3 hrs. 15 m. 15 sec.
6. £1 7*s.* 6*d.* : £3 15*s.*
7. 3 a. 1 r. 20 p. : 5 a. 2 r. 10 p.
8. £15 15*s.* : £16 16*s.*

RULE OF THREE.

131. The practical application of the foregoing considerations is called the Rule of Three, because three out of four proportionals are used to find the fourth (129 *b*). The rule is applicable in cases in which proportionals occur as with reference to weights and values—quantities and prices—wages and time—debts and part-payments, &c.

The UNITARY METHOD, by which such calculations can also be made, is explained at the end of this chapter (page 85).

If the value of 13 yds. is £2 12*s.*, what are 18 yds. worth?

Since 13 yds., 18 yds. have the same ratio as the two values 52*s.* and value (shillings) sought, we write the proportionals—

Material. *Answer.*
13 yds. : 18 yds. :: 52*s.* : (*shillings ?*)
(by 130) 13 : 18 :: 52*s.* : (*shillings ?*)
worked out by (129*b*).

$$s.\ \overset{4}{\cancel{52}} \times 18 = \text{product of m}^{\text{s}} = \frac{\text{product of ex}^{\text{s}}}{\underset{1}{\cancel{13}}} = 72s.\ (\textit{fourth term}).$$

132. It is to be observed in the working out

(*a*) that the *third term* is the *material* out *of which* the fourth term—the *answer—is made*

(*b*) that this material is multiplied by the second term, *an abstract number*, to obtain the "product of means," which is also the "product of extremes" (128)

(*c*) that out of the "product of extremes" (4th term × 1st term) the first term, *an abstract number*, is taken away by division (29) and the factor remaining is *the fourth term or answer.* $\frac{4\text{th} \times \cancel{1\text{st}}}{\cancel{1\text{st}}} = 4\text{th}.$

133. The following directions will serve as a guide in working by Rule of Three.

(*a*) Place in the *third* term the *material* out *of which* the *answer is* to be *made.* (*b*) Consider whether the answer (4th term) will be greater or less than the third term. (*c*) Arrange the other terms so that the multiplying No. (2nd term) and the dividing No. (1st term) will bring out the answer, greater or less, as required.

Ex. LXXXIII.

1. If 36 bushels cost £12, how many bushels may be bought for £84?
2. If 180 gallons cost £90, how many may be bought for £100?
3. How many acres of land may be bought for £1200, at £400 for 80 acres?
4. If 15 lbs. of sugar cost 6*s.*, how many lbs. may be bought for 120*s.*?
5. What must be given for 18 cwt., if 27 cwt. cost £36?
6. At 12*s.* for 6 yds., how many yds. could be bought for 120*s.*?

134. In some cases, quantities are *inversely* proportional to each other, as twice the number of men will take half the time—twice the rate of speed will require only half the time; half the allowance will supply double the number of men, &c., and in arranging the first and second terms in such cases, the multiplying and dividing numbers must be so placed as to give a greater or less answer, as required.

If, when travelling 5 hrs. a day, a journey takes 26 days, how many days will it occupy when travelling 10 hrs. a day?

			Material.	*Answer.*
	10 hrs. :	5 hrs. ::	26 dys. :	(*dys. reqd.?*)
expressing the ratio of 10 hrs. to 5 hrs. by abstract numbers.	10 :	5 ::	26 dys. :	„

$$\text{dys. } \frac{26 \times \overset{1}{\cancel{5}}}{\underset{2}{\cancel{10}}} = \frac{26}{2} = 13 \text{ dys.}$$

Ex. LXXXIV.

1. If 70 workmen can build a house in 200 days, how many can do the same amount of building in 25 days?

2. If a person travelling 12 hrs. a day complete a journey in 6 days, how many days will the return journey occupy, travelling 8 hrs. a day?

3. If 500 men have provisions for 6 weeks, how many weeks would the same provisions last 800 men?

4. If 17 horses will eat up a rick of hay in 15 days, how many horses would eat it up in 115 days?

5. How much land at 112*s.* an acre would be equal in value to 50 acres at 100*s.* an acre?

6. If 13 men can build a wall in 112 days, how many men would build a similar wall in 54 days?

135. Examples in which occur proportionals, with reference to weight and value; measure and value; quantity and value; number and value; time and money; distance and money, &c.

FORMS OF STATING.

				Material.		*Answer.*
weight	:	weight	::	value	:	(*value?*)
or value	:	value	::	weight	:	(*weight?*)
measure	:	measure	::	value	:	(*value?*)
or value	:	value	::	measure	:	(*measure?*)
&c.				&c.		

136. N.B.—The third term may be multiplied and divided in any form found most convenient.

The *first and second terms* must be reduced to the *same denomination*, and the *abstract numbers*, which then express their ratio, *are used in mult*

If 2 cwt. 1 qr. 8 lbs. of sugar cost £4 6*s*. 4*d*., what must be given for 5 cwt. 2 qrs. 14 lbs.?

	cwt. qrs. lbs.		cwt. qrs. lbs.		Material. £ *s*. *d*.		
(40)	2 1 8	:	5 2 14	::	4 6 4	:	(*cost*?)
	224		560		20		
	28		56		86		
	8		14		12		
	260 (lbs.)		630 (lbs.)		1036*d*.		

$$\frac{\overset{518}{d.\,1036} \times 63\cancel{0}}{\underset{13}{26\cancel{0}}} = \begin{array}{r} 518d. \\ 63 \\ \hline 1554 \\ 3108 \\ \hline 13\,)\,32634d. \\ 12\,)\,2510\tfrac{4}{13}d. \\ 20\,)\,209s.\;2\tfrac{4}{13}d. \\ \hline £10\;9s.\;2\tfrac{4}{13}d. \\ \hline \end{array}$$

Ex. LXXXV.

1. If 35 lbs. of meat cost £1 17*s*. 6*d*.; what must be given for 75 lbs.?
2. If 20 sheep are worth £100 17*s*.; what is the value of 318 sheep?
3. If 27 bushels of oats cost £15 4*s*. 6*d*.; how many bushels may be bought for £137 0*s*. 6*d*.?
4. If 38 c. 1 qr. 7 lbs. (40) cost £52 7*s*. 6*d*.; what will 18 cwt. cost?
5. What weight may be bought for 100 guineas, if 14 cwt. 1 qr. 16 lbs. cost £100?
6. How many gallons of wine may be bought for £136 5*s*., when 15 gallons cost £15 6*s*. 8*d*.?
7. Find the value of one-eighth of 17 cwt. 2 qrs. 14 lbs. (40), at £3 17*s*. 8*d*. for 1 cwt. 1 qr. 4 lbs. 8 ozs. (40.)
8. What weight may be bought for 100½ guineas, if 13 cwt. 1 qr. 12 lbs. cost £51 14*s*. 6*d*.?
9. Add together 3 cwt. 1 qr. 4 lbs. and 1 qr. 14 lbs. 5 ozs., and find the value of the whole at 12*s*. 10½*d*. for 3 qrs. 16 lbs. 4 ozs.
10. If 15¼ yds. cost £4 16*s*. 10½*d*., what must be given for 20 yds.?

11. At 5*s.* 6*d.* per oz., what weight of silver can be bought for £10 10*s.*?

12. If £3 14*s.* 6*d.* be paid for the carriage of 2 cwt. 1 qr. 4 lbs. (40), what must be paid for 4 barrels, each weighing 1 cwt. 3 qrs. 19 lbs. (40)?

13. If a person spends £1 18*s.* 6*d.* in 13 days, how much will he spend at the same rate in a year?

14. If a man's wages are at the rate of 19*s.* 10*d.* for 3 dys., how much will he receive for work done in 1 year?

15. If $3\frac{3}{4}$ shares in a mine cost £$141\frac{5}{8}$, what must be given for $18\frac{1}{8}$ shares?

16. If $3\frac{1}{8}$ lb. Av. cost $17\frac{1}{6}$*s.*, what must be given for $13\frac{1}{3}$ cwt.

136. Examples in which occur proportionals, with reference to money taxed and tax—income, income tax—rental from property, property tax (paid by owner)—rental from land, land tax (paid by owner)—rental of house, inhabited house duty (paid by occupier).

FORMS OF STATING.

					Material.		*Answer.*
(*a*)	money taxed	:	money taxed	::	tax	:	(*tax?*)
(*b*)	tax	:	tax	::	money taxed	:	(*money taxed?*)

17. If the land-tax be charged at the rate of £12 15*s.* on every £100 received as rent; what will be charged on a rental of £8500?

18. If the income-tax be $7\frac{1}{2}$*d.* on every £, how much will be paid out of an income of £540 10*s.*?

19. What is the income of one who pays £15 18*s.* 10*d.* as income-tax, at the rate of $6\frac{1}{2}$*d.* in the £?

20. If the income-tax be 8*d.* on the £, what tax will be paid by one whose income is £540 10*s.*?

21. If the property-tax be charged at the rate of $5\frac{1}{2}$*d.* on the £, what will be levied on £1500 17*s.* 6*d.*?

22. If a person pays £13 17*s.* 6*d.* as inhabited house duty, at the rate of 9*d.* in the £, on what amount of rental does he pay?

137. In towns and parishes, to meet local expenses, rates are levied according to the assessed rental, which is usually about two-thirds of the rent actually paid.

FORMS OF STATING.

					Material.		*Answer.*
(*a*)	rent	:	rent	::	rate	:	(*rate?*)
(*b*)	rate	:	rate	::	rent	:	(*rent?*)

23. A person pays a poor rate of £15 18*s.* 6*d.*, and the rate is at 2*s.* 10*d.* on the £, on what rental was it levied?

24. The rents of a parish taken all together make up £15,075, and £562 18*s.* 6*d.* must be obtained for the support of the poor, how much on the £ will be levied?

25. A general-improvement-rate at 2*s.* 6*d.* in the £ is to be collected; how much will a person be required to pay on his assessed rental of £179 18*s.* 6*d.*?

26. At 7½*d.* poor-rate on the £, what is the rent of one who has to pay £1 17*s.* 6*d.*?

138. Examples in which occur proportionals with reference to debt and part-payments. In such examples *money paid* is variously called *property*, *assets*, *money received*, *dividend*.

FORMS OF STATING.

					Material.		*Answer.*
(*a*)	debt	:	debt	::	money paid	:	(*money paid?*)
(*b*)	money paid	:	money paid	::	debt	:	(*debt?*)

27. A bankrupt owes £5276 14*s.*, and his assets are in value £1754 14*s.*; how much will he pay in the £?

28. A bankrupt has property to the value of £890 10*s.*, and he is able to pay 11*s.* 6*d.* in the £; what do his debts amount to?

29. A bankrupt pays 5*s.* 6*d.* in the £, and the payment he can make amounts to £570 5*s.*; how much does he owe?

30. A bankrupt owes £12,785, and his effects realise in sale £2785; how much will he pay in the £?

31. How much will a creditor *lose* out of his claim for £275 10*s.*, if a dividend of 12*s.* 6*d.* in the £ is paid?

32. A bankrupt pays 11*s.* 6*d.* in the £; how much will be *lost* on a debt of £100 13*s.* 4*d.*?

33. A bankrupt pays £1276, at the rate of 8*s.* 10*d.* in the £, find the amount of his debts.

139 (*a*). In examples in which reduct^{n} of first and second terms to the same name is best worked out fractionally, it is often enough to *indicate the changes*, and not to multiply up until *third* × *sec*^{d} ÷ *first*.

(*b*) The changes by which the material is brought to the same name as the answer should only be *indicated* at first.

If $\frac{2}{5}$ of a ton cost £2 16*s.*, what must be given for 3 qrs. 1 lb.?

$$\overbrace{\frac{2t}{5}\times\frac{20}{1}\times\frac{4}{1}}^{\text{qrs.}} \; : \; \overset{\text{qrs.}}{3\tfrac{1}{28}} \; :: \; \overbrace{\frac{56s.}{1}\times\frac{1}{20}}^{\text{£.}} \; : \; (\text{£ ?})$$

$$\overbrace{\frac{\overset{2}{\cancel{56}}}{1}\times\frac{1}{\underset{4}{\cancel{20}}}}^{\text{£}}\times\frac{\overset{17}{\cancel{85}}}{\underset{1}{\cancel{28}}}\times\frac{\overset{1}{\cancel{5}}}{\cancel{2}}\times\frac{1}{\underset{4}{\cancel{20}}}\times\frac{1}{4}=\overset{\text{£}}{\frac{17}{64}}= \begin{array}{rrrr} & \text{£} & s. & d. \\ 8) & 17 & 0 & 0 \\ \hline 8) & 2 & 2 & 6 \\ \hline & & 5 & 3\tfrac{3}{4} \end{array}$$

If $3\frac{2}{5}$ E. ells cost £7 12*s.* 2*d.*, what must be paid for $3\frac{3}{8}$ yds.?

$$\overbrace{\frac{17\text{ E. e.}}{5}\times\frac{5}{1}}^{\text{qrs.}} \; : \; \overbrace{\frac{27\text{ yds.}}{8}\times\frac{4}{1}}^{\text{qrs.}} \; :: \; \overbrace{152\tfrac{1}{6}s.\times\tfrac{1}{20}}^{\text{£}}$$

$$\overbrace{\frac{913}{\underset{2}{\cancel{6}}}\times\frac{1}{20}}^{\text{£}}\times\frac{\overset{9}{\cancel{27}}}{\underset{2}{\cancel{8}}}\times\frac{\overset{1}{\cancel{4}}}{1}\times\frac{\cancel{5}}{17}\times\frac{1}{\cancel{5}}=\frac{\text{£}8217}{1360}\,(81\,b)=\text{£}6\quad 0s.\quad 1\tfrac{1}{17}d.$$

Ex. LXXXVI.

1. If $9\frac{1}{3}$ oz. Av. cost $8\frac{1}{48}$*s.*, what will $17\frac{1}{2}$ lbs. cost?
2. If $\frac{3}{5}$ of $\frac{5}{9}$ of a ship be worth £12,000, what will $\frac{2}{7}$ of $\frac{14}{15}$ of the ship cost?
3. If $1\frac{1}{6}$ cwt. cost $3\frac{3}{4}$ guineas, what must be given for $16\frac{1}{3}$ lbs.?
4. If $3\frac{3}{4}$ yds. cost 17*s.* 6*d.*, what must be given for $3\frac{3}{4}$ E. ells?
5. At £$\frac{1}{8}$ for $3\frac{3}{4}$ dwts., what will be the value of $1\frac{7}{8}$ oz.?
6. If $8\frac{5}{8}$ cwt. cost £22 17*s.* 6*d.*, what must be given for 3 tons $14\frac{3}{4}$ cwt?

THE UNITARY METHOD.

Questions in Rule of Three can be reasoned out by the Unitary Method, which the following examples will illustrate.

If the value of 13 yds. is £2 12*s.*, what is the value of 18 yds.?

13 yds. are worth 52*s.*

1 yd. is „ $\frac{52s.}{13}=4s.$

18 yds. are „ $4s.\times 18=72s.=\text{£}3\ 12s.$

If when travelling 5 hrs. a day, a journey takes 26 days, how many days will it take when travelling 10 hrs. a day?

5 hrs. a day take 26 dys.

1 hr. „ will take 26 dys. × 5.

10 hrs. „ „ dys. $\frac{26\times 5}{10}=13$ dys.

If 2 c. 1 qr. 8 lbs. cost £4 6*s.* 4*d.*, what must be given for 5 c. 2 qrs. 14 lbs.?

260 lbs. cost 1036*d.*

1 lb. costs $\frac{1036d.}{260}$.

630 lbs. cost $\frac{1036d.}{260} \times \frac{630}{1} = £10\ 9s.\ 2\frac{4}{13}d.$

At 7*d.* poor rate in the £, what is the rent on which is paid a poor rate of £1 15*s.*?

7*d.* is the rate on £1 rent.
1*d.* „ „ £$\frac{1}{7}$ rent.
420*d.* „ „ £$\frac{1}{7} \times 420 = £60$.

Directions for working:—Observe that the question consists of two parts; one containing two connected terms, the other a single term which is asked about. Write down in a line the two connected terms, using with them words which show their relation, and *placing the term like the required answer on the right-hand side;* then, by multiplication or division get on the right-hand side an equivalent of 1 *on the left*, and thence obtain required answer.

Examples for practice will be found in LXXXIII., LXXXIV., and LXXXV, and in questions on Stocks, &c.

COMPOUND PROPORTION.

140. In some questions of proportion, the result depends on several circumstances, and is determined with reference to two or more ratios; such examples are said to be instances of Compound Proportion.

If 10 men can reap a field of 50 acres in 4 dys., how many men will be required to reap 80 acres in 8 dys.?

In this example, 50 acres and 80 acres, also 4 days and 8 days will give ratios by which the answer will be found. As to the former, the proportion is direct, a greater number of acres requiring a greater number of men; as to the latter, the proportion is inverse, for a longer time will require a proportionately smaller number of men; hence, placing 10 men in the 3rd term as the material, from which the answer is to be made, we must state thus and work out as in simple proportion.

50 ac. : 80 ac. :: 10 men : (*men?*)
8 dys. : 4 dys.

$$\text{men } \frac{\overset{2}{\cancel{10}} \times \overset{\cancel{10}}{\cancel{80}} \times 4}{\underset{5}{\cancel{50}} \times 8} = 8 \text{ men.}$$

Ex. LXXXVII.

1. If 8 men can reap 16 acres in 2 days, how many acres can 9 men reap in 14 days?

2. If £12 10*s.* will pay 12 labourers for 6 days, how much money will pay 18 labourers for 5 days?

3. How many men can dig 4 acres in 3 days, if 18 men can dig 16 acres in 4 days?

4. If 9 cows be kept 12 days for £10 15*s.*, how many cows could be kept 5 days for £5 12*s.*?

5. If 17 persons spend £110 in 3 months, how long will £53 5*s.* last 4 persons, if they spend at the same rate?

6. If 3 horses eat 187 lbs. of oats in 4 days, what quantity will 5 horses eat in 17 days?

7. If the wages of 8 men for 5 days' work be £9 5*s.*, what number of men will receive £37 15*s.* for 16 days' work?

8. How many reams of paper will be required to print 4000 copies of a book of 12 sheets, if 66 reams are required for 5000 copies of a book of 10 sheets?

9. If the expenses of 8 persons for 8 wks. 3 dys. are £65 17*s.* 6*d.*, what sum will be spent by 13 persons in 3 wks. 5 dys., at the same rate?

10. If 5 men earn £7 13*s.* 6*d.* in 5½ dys., how much will 13 men earn in 17¼ dys.?

11. If 15 men mow 60 acres of grass in 6 dys., how many men will be required to mow 120 a. 2 r. 10 p. in 4 dys.?

12. If 17 horses plough 17½ acres in 6½ dys., how many horses will be required to plough 14¾ ac. in 5¼ dys.?

13. If 8 compositors set up 6 sheets in 1 day, in how many days will 11 compositors set up 110 sheets?

14. If a man travels 65 miles in 3 days, walking 6¼ hrs. every day; how long will it take him to walk 158 miles, walking 5¾ hrs. per day?

15. If 16 men build 30 rods of wall in 21 days, working 10 hrs. a day, how many hrs. a day must 15 men work to build 33 rods in 19 dys.?

16. If 9 men can dig a trench 58 ft. long, 4 ft. broad, and 3 ft. 6 in. deep, in 12 hrs., how many will be required to dig a trench 80 ft. long, 3½ ft. broad, and 2¼ ft. deep?

17. How far should 17 c. 3 qrs. be carried for £15 10*s.*, if 18 c. 1 qr. are carried 18 miles for £19 17*s.* 6*d.*?

18. If 7 men build a wall 30¼ yds. long, and 12¾ ft. high, in 18 dys. of 8 hrs. each, how many men will be required to build a wall 18¾ yds. long, 10½ ft. high, in 16¼ dys. of 16¼ hrs. each?

19. If 100 men in 4 dys. of 12 hrs. each can dig a trench 150 yds. long, 3 wide, and 5 deep, in how many days of 7 hrs. each will 180 men dig a trench 210 yds. long, 4 wide, and 3 deep?

20. If 8 men complete a piece of work in 15¼ dys., working 6¾ hrs. a day, how many would do the same work in 17⅛ dys., working 5¾ hrs. a day?

MISCELLANEOUS EXAMPLES (LXXVI—LXXXVII.).

The following should be noted.

(*a*) cost + gain = selling price.
(*b*) cost − loss = selling price.
(*c*) selling price − cost = gain.
(*d*) cost − selling price = loss.
(*e*) income = money spent + money saved.
(*f*) money spent = income − money saved.
(*g*) money saved = income − money spent.
(*h*) part of £, &c., paid as tax. remainder of £, &c., is net or clear.

A. In all cases the sums of money added or subtracted, must belong to the *same* weight, *same* time, *same* quantity, &c.

B. When the cost, gain, loss, &c., belonging to *a certain quantity* is known, the cost, gain, loss, &c., belonging to *any other quantity* can be found either by Rule of Three, Multiplication, or Division.

1. If an estate produce £450 a year, and the land-tax be assessed at 2*s.* 8*d.* in the £, what rent will remain clear to the owner? (*h.*)

2. If a person lend me £750 for 9 months, how long should I lend him £1550 to return his kindness? 134.

3. Find the value of 14 cwt. 1 qr. 14 lbs. (40) of sugar, at £2 7*s.* 6*d.* for 1 cwt. 1 qr. 18 lbs. (40.)

4. If a bankrupt pay £1500 out of a debt of £2100 7*s.* 6*d.*, how much does he pay in the £? 138.

5. How many dollars, each 4*s.* 6*d.*, are equal in value to 355 pieces, each 6*s.* 6*d.*? 134.

6. Bought sugar at 6½*d.* per lb., and sold it at 8¾*d.* per lb., how much did I gain on 1 cwt. 1 qr. 18½ lbs.? (40.) c. B.

7. If I pay £7 7*s.* 6*d.* for income-tax yearly at 6½*d.* in the £, what is my income? 136.

8. If the carriage of 4 cwt. 3 qrs. for 175 miles be 11*s.* 6*d.*, what would be the charge for 1 ton 15 cwt.?

9. Find the value of 5 chests of tea, each 3 qrs. 14 lbs. (40) at £2 3*s.* 6*d.* for $9\frac{1}{2}$ lbs.

10. Bought 215 yds. of cloth for £185 17*s.* 6*d.*, 40 yds. were lost, at how much per yd. must the remainder be sold to gain 15 guineas upon the whole? (a.) B.

11. If a person spend £3 17*s.* 6*d.* in 3 dys., and find that £150 15*s.* 8*d.* remain at the year's end, what is his yearly income? B. (e.) A.

12. If the tax on a house rated at £76 10*s.* be £12 12*s.*, what would it be on one rated at £100? 136.

13. Find the value of 7 parcels, each weighing 1 cwt. 1 qr. 18 lbs. (40), at £1 5*s.* 6*d.* per cwt.

14. How many English ells may be bought for 100 guineas, if $5\frac{1}{2}$ yds. cost £2 15*s.*? 139 (*b.*)

15. If £4 5*s.* 6*d.* be lost on the sale of 12 cwt. 3 qrs. 7 lbs. (40) of coffee, on what weight would £100 be lost?

16. A bankrupt owes £2500, and his property is worth only £937 10*s.*, how much on the £ will his creditors receive? 138 (*a.*)

17. If 4 cwt. 1 qr. 15 lbs. (40) be bought for 40 guineas and sold for £52 17*s.* 6*d.*, what is the rate of gain per cwt.? *c.* B.

18. If 19 men dig a trench in 16 days, in what time will 15 men do it? 134.

19. At a cost of 4*s.* 6*d.* per lb., what must 3 cwt. 1 qr. 8 lbs. (40) be sold for to gain £6 15*s.* 6*d.*? B. (*a.*)

20. If I wish to lay by £150 every year out of an income of £750 17*s.* 6*d.*, how much may I spend every day? *f* B.

21. If $7\frac{3}{4}$ yds. of silk cost £7 12*s.* 6*d.*, what must be given for $23\frac{1}{3}$ yds.? 139.

22. If $3\frac{1}{3}$ cwt. cost £25 8*s.*, what will $6\frac{1}{3}$ lbs. cost? 139.

23. If $\frac{2}{3}$ of an English ell cost £5 $5\frac{3}{4}$*s.*, what must be given for $3\frac{1}{3}$ times $2\frac{1}{2}$ yds.? 139 (*b.*)

24. If $15\frac{1}{4}$ lbs. cost £$\frac{19}{20}$, what will $\frac{1}{2}$ of $\frac{3}{4}$ of $5\frac{1}{7}$ qrs. cost? 139.

25. What must be given for $7\frac{1}{2}$ pieces of cloth, each $15\frac{1}{8}$ yds., at £2 7*s.* 6*d.* for $3\frac{1}{4}$ English ells? 139.

26. If $12\frac{1}{7}$ cwt. be carried $\frac{5}{16}$ of a mile for $\frac{1}{8}$ of a £, what would be the carriage of $13\frac{5}{7}$ lbs. for $8\frac{1}{3}$ miles? 140, 139.

27. If $\frac{3}{17}$ of an estate be worth £8000, what is the value of $\frac{1}{7}$ of $\frac{1}{4}$ of it?

28. If $12\frac{5}{6}$*s.* are spent every day, to what sum will it amount in a year?

29. If an estate produce £$17,536\frac{3}{5}$ in a year, and the land-tax be charged at the rate of $3\frac{5}{6}$*s.* on every £, what is the clear annual income? (*h.*)

30. How much will a man, whose debts amount to £$12,357\frac{3}{20}$, pay in the £ out of his property which sells for £$8429\frac{5}{9}$? 138 (*a.*)

31. Find by Practice the cost of 15 tons 12 cwt. 2 qrs. at £1 5*s.* 6*d.* per ton.

32. What will be the dividend on £1525 10*s.* at 5*s.* $6\frac{1}{2}$*d.* per £? (Practice.)

33. If a person pays as income-tax £25 7*s*. 6*d*. when the tax is at 6*d*. in the £, what will he pay when the tax is at 8½*d*. in the £?

34. If 3 men earn £8 in 8 days, how much should 15 men receive for 6⅔ days' work? 140.

35. Find by Practice the rent of 144 ac. 2 r. 15½ p., at £2 15*s*. 6*d*. per acre.

36. What is the tax on £1000 17*s*. 6*d*., at 4*s*. 6¾*d*. in the £? (by Practice.)

37. A man having paid his income-tax at the rate of 6*d*. in the £, has £287 10*s*. remaining, what was his full income?

38. If a man's income is £375, and there remains to him £365 12*s*. 6*d*. after his tax is paid, what is the rate of tax on the £?

39. Starting with the value at 1*s*. each, find the value of 17,346 articles at 18*s*. 9¾*d*. each. 125.

40. What will be the value of 1½ tons, at the rate of £4 6*s*. 10*d*. for 1 cwt. 1 qr. 14 lbs.?

141. Calculations in Interest, Discount, Insurance, Commission, Stocks, and Profit and Loss, are worked out by Rule of Three, and it will be sufficient to indicate the forms of stating, by which the required results are to be obtained.

142. Here, observe that two connected terms as "at 5*s*. 6*d*. for 7 yds." may enter into two different statings. The arrow head is used to denote the third term (Material 132) in a set of proportionals.

$$\begin{tikzcd}[column sep=small] 5s.\ 6d. \arrow[rr, bend right] & \text{for} & 7\ \text{yds.}\end{tikzcd}$$

(*a*) 5*s*. 6*d*. : any other money : : 7 yds. : (*yds.?*)

$$\begin{tikzcd}[column sep=small] 5s.\ 6d. & \text{for} & 7\ \text{yds.} \arrow[ll, bend left]\end{tikzcd}$$

(*b*) 7 yds. : any other yds. : : 5*s*. 6*d*. : (*money?*)

The result being from (*a*) the number of yards belonging to any money placed in the second term.
(*b*) the money belonging to any number of yards placed in the second term.

143. And it will be useful to mark the particular things which must be known, in order to obtain a particular result by proportionals, as—

(*a*) to find quantity &c. belonging to a particular money;

quantity &c. belonging to some other money must be known;

(*b*) to find money belonging to a particular quantity &c. *money belonging to some other quantity &c. must be known ;*

hence the *result to be found, guides* us *in searching for* the particular *terms*, which will give a set of proportionals.

INTEREST.

144. Interest is the money paid for the *loan* of a certain sum of money. The money lent is called the Principal. The Rate of Interest is always given at a certain payment for the loan of £100 for 1 year. *Principal and* its *Interest* for any particular time, *added* together, *give Amount.*

145. When the Principal remains the same year after year, the money is said to be out at Simple Interest. When at the end of every year, or at any times agreed upon, the Interest, instead of being paid to the lender, is added to the Principal, and thus the Principal is periodically increased, the money is said to be out at Compound Interest.

To find the Simple Interest on a given Principal at a given rate per cent. per annum; that is, at a given yearly rate of interest on every £100 lent, state

Principal : Principal : : Interest : (*Interest ?*)

Find the simple interest on £565 10*s.* for 3 years, at $2\frac{1}{2}$ (I.) per cent. (£100 P) per annum (1 year).

As 3 years' int. is req^{d}., place 3 years' int. 1 *yr's. int.* × 3 on £100, as material in third term.

$$\pounds 100 \text{ P} : \pounds 565\tfrac{1}{2} \text{ P} :: \overset{\text{3 years' Interest.}}{\pounds 2\tfrac{1}{2} \times 3} : (\textit{int. ?})$$

$$\frac{\pounds\cancel{5}}{2} \times \frac{3}{1} \times \underset{20}{\frac{1131}{2}} \times \frac{1}{\cancel{100}} = \frac{\pounds 3393}{80} = \pounds 42\ 8s.\ 3d.$$

Compare, with working above, the formula $\frac{\text{P} \times \text{R} \times \text{Y}}{100} = \text{Int.}$

Ex. LXXXVIII. Find at simple interest

1. Interest on £350 for 4 years, at 5 per cent.
2. Interest on £575 for 8 years, at 4 per cent.
3. Amount of £1525 for 2 years, at 2 per cent.
4. Amount of £2355 for $3\frac{1}{4}$ years, at $2\frac{1}{2}$ per cent.
5. Interest on £545 10*s.* for 1 year, at 5 per cent.
6. Interest* on £355 15*s.* for $5\frac{1}{2}$ years, at $4\frac{1}{2}$ per cent.
7. Interest on £150 17*s.* 6*d.* for 2 years, at $2\frac{1}{4}$ per cent.
8. Amount of £500 15*s.* for $3\frac{1}{4}$ years, at $3\frac{1}{4}$ per cent.
9. Interest on £213 13*s.* 4*d.* for 6 years, at $5\frac{1}{2}$ per cent.
10. Interest on £130 for $3\frac{1}{4}$ years, at 5 per cent.

146. Should the time be given in such a form that months and days occur, it must be expressed in *years and fraction of a year* (102) that 1 YR.'s INT. × NO. OF YRS. may be the THIRD TERM, as in the preceding example.

Find the simple interest on £235 17*s.* 6*d.* for 3 yrs. 8 m., at 4 per cent. per annum.

(3 yrs. 8 m. $= 3\frac{8}{12}$ yrs.
$= 3\frac{2}{3}$ yrs.

$\text{£}100 : \text{£}235\frac{7}{8} :: \overbrace{\text{£}4 \times 3\frac{2}{3}} : (\textit{Int.}?)$

Find the simple interest on £245 14*s.* 6*d.* from March 4, 1881, to June 27, 1883, at 5 per cent. per annum.

Time = 2 yrs. 115 dys.
$= 2\frac{115}{365}$ yrs.
$= 2\frac{23}{73}$ yrs.

$\text{£}100 : \text{£}245\frac{29}{40} :: \overbrace{\text{£}5 \times 2\frac{23}{73}} : (\textit{Int.}?)$

Ex. LXXXIX. Find at simple interest

1. Interest on £500 for 3 yrs. 5 m., at $3\frac{1}{2}$ per cent.
2. Interest on £730 for 4 yrs. 35 dys., at 3 per cent.
3. Interest on £1250 10*s.* for 4 yrs. 7 m. at $5\frac{1}{2}$ per cent.
4. Amount on £1255 15*s.* for 6 yrs. 315 d. at 4 per cent.
5. Interest on £600 from Feb. 1, 1882, to Mar. 8, 1883, at 6 per cent.
6. Interest on £1500 from Mar. 10, 1868, to May 4, 1873, at 5 per cent.
7. Interest of £525 from 1 Jan., 1882, to 25 June, 1883, at $1\frac{1}{2}$ per cent.
8. Amount on £225 for 3 yrs. 5 m. at $5\frac{1}{2}$ per cent. per annum.

COMPOUND INTEREST.

147. To find the Compound Interest on a given sum for a given time, at a given rate, work out one year's Interest and add it to its Principal—then find a year's Interest on that increased Principal, and adding it, obtain the Principal on which Interest will be charged for the following year—and so on, until the particular time given in the question has been reached.

148. To find one year's Interest, observe what part the given interest is of its principal, and keep 100 as the denominator; thus at 5 per cent. Interest = $\frac{5}{100}$ of its Principal; at 4 per cent. Interest = $\frac{4}{100}$; at 2½ per cent. Interest = $\frac{2\cdot5}{100}$. Work by decimals, and in dividing by 100, remove figures two places to the right (106).

What is the Compound Interest on £125 16*s.* 6*d.* for 2 yrs. at 2½ per cent.?

£125 16*s.* 6*d.* = £125·825. One yr's. int. = $\frac{2\cdot5}{100}$ of its principal.

First Principal . .	£125·825 × 2·5 ÷ 100
Int. for 1st yr. . .	3·145625
Second Principal .	128·970625 × 2·5 ÷ 10
Int. for 2nd yr. . .	3·224265625
Third Principal . .	132·194890625
First Principal . .	125·825
Interest	6·369890625
	20
	7·3978125~~00~~
£6 7*s.* 4*d.* int.	12·
	4·77375~~00~~

Ex. XC.

1. Find the Compound Interest on £345 15*s.* at 4 per cent. for two years.
2. Find the Compound Interest on £150 for 3 yrs. at 5 per cent.
3. Find the difference between the Simple and Compound Interest on £125 10*s.* for 3 yrs. at 5 per cent.
4. Find the Compound Interest on £350 12*s.* 6*d.* for 3 yrs. at 4 per cent.
5. Find the Compound Interest on £500 for 2 yrs. at 10 per cent.
6. What is the difference between the Simple and Compound Interest on £250 for 2 yrs. at 2½ per cent.?

149. In some questions which have to be worked out by Rule of Three, ONE OR MORE OF THE TERMS *necessary to make a set of proportionals are not given, but* HAVE TO BE MADE *out of materials found in the question*, and this is so in the three cases, which occur as variations to that in Simple Interest already considered, in which Principal, Rate per Cent., and Time are given, to find Interest.

Case I. Given Principal, Amount and Rate, to find Time.
„ II. „ Rate, Time, and Interest, „ Prin^{l}.
„ III. „ Principal, Time, and Interest, „ Rate.

CASE I. In what time will £200 be £250 at 4 p. c. p. ann.?

£250 amount – £200 principal = £50 int. The question now stands, "In what time will £200 gain £50 int."? and to find the time, which belongs to that int., we must know the time which belongs to some other int.; then finding 1 *yr.'s* int. on £200 (45) viz. £8 int. for 1 yr.

£8 int. : £50 int. :: 1 yr. : (yrs ?).

In this stating, 1 *year* and 1 *year's Interest on given Principal* supply two terms.

Ex. XCI.

1. In what time will £250 amount to £350, at 5 per cent. per annum?
2. In what time will £350 gain £42 interest at 4 per cent. per annum?
3. In what time will £850 amount to £1147 10*s*., at 5 p. c. per ann.?
4. In what time will the interest on £1500 amount to £337 10*s*., at 2½ per cent. per annum?

CASE II. What sum of money put to interest for 4 yrs., at 5 per cent., will amount to £300?

Answer req^{d} is principal, therefore 3rd term (Material) must be principal, and a term must be found to correspond to the £300, which is *a* 4 *yr.'s amount*. Taking £100 as a principal, its 4 *yr.'s amount* is £120.

£120 amt. for 4 yrs. : £300 amt. for 4 yrs. £100 pr. : (*pr.* ?)

In this stating £100 *principal* and *its amount for given time* supply two terms.

Ex. XCII.

1. What sum of money will amount to £1260 in 2 yrs. at 6 per cent ?
2. What money will amount to £2673 2*s*. 6*d*. in 3½ yrs. at 5 per cent. ?
3. What money will amount to £954 2*s*. 6*d*. in 3½ yrs. at 3½ per cent. ?
4. What principal will amount to £1634 7*s*. 6*d*. in 8¾ yrs. at 7¼ p. c. ?

Case III. At what rate p. c. do £500 become £750 in 5 yrs.?

The question asks for the interest on £100 for 1 year; then, £750 amount – £500 principal = £250 interest in 5 yrs., and £250 ÷ 5 = £50 interest for 1 year on the principal £500.

$\longleftarrow$

£500 pr. : £100 pr : : £50 int. for 1 yr. : (1 *yr.'s int.*)

In this stating the *given principal* and *its one year's interest* supply two terms.

Ex. XCIII.

1. At what rate per cent. will £1125 amount to £1260 in 4 yrs. ?
2. At what rate per cent. will £70 amount to £80 in 4 yrs. ?
3. At what rate p. c. will £2275 amount to £2673 2*s*. 6*d*. in 3½ yrs. ?
4. At what rate per cent. will £575 amount to £776 5*s*. in 8¾ yrs. ?

150. Discount is a deduction allowed for the payment of money before the date at which it is due by arrangement.

151. Apart from any consideration of time, Discount is simply a reduction of debt, because immediate or cash payment is made, as 1*d*. off every 1*s*., *i.e.* $\frac{1}{12}$th off; 1*s*. off every £, *i.e.* $\frac{1}{20}$th off; 5 per cent. off, *i.e.* $\frac{1}{20}$th.

152. Let us suppose that A buys of B £2000 worth of silk on condition that payment is to be made at the end of 6 months; now, to pay at once would be to lose 6 months' interest on £2000; but the debt could be reduced and so adjusted, that the *present payment with* ITS *interest*, would just come up to £2000 at the end of 6 months; but this is never done. Wholesale houses of business give credit for three months, &c., according to their published terms; and, in most cases, when payment is not made at the appointed time, a Bill, termed an Acceptance or a Promissory Note, is drawn out and forwarded for acceptance, which Bill is passed into the Banking account, and discounted.

153. FORM OF ACCEPTANCE.

£100, due April 4, 1883.
London, 1*st January*, 1883.
Three months after date pay to our (or my) order, the sum of one hundred pounds sterling, value received.
S. WARD & Co.
To Messrs. Smith & Co.,
Belfast.

FORM OF PROMISSORY NOTE.

£50, due 4th April, 1883.
London, 1*st January*, 1883.
Three months after date, I (or we) promise to pay Mr. James Robinson, or order, the sum of fifty pounds sterling, for value received.
Payable at Bank of England.
JOHN WHITE.

Chichester, 1st June, 1883.

£500. Three months after date pay to Messrs. Hamilton & Co., or Order, the sum of Five hundred Pounds sterling, for value received.

SPENCER & Co.

To Messrs. Robinson & Sons.

In this case Messrs. Spencer & Co. call upon Messrs. Robinson & Sons to pay on their account to Messrs. Hamilton & Co. Five hundred Pounds, on the 4th Sept. (three months with three days' grace), and Messrs. Robinson & Sons *accept* to do so *by writing to that effect across the bill.*

If on this Bill, money were advanced by a Banker or Bill Discounter, say on 6th July, a deduction of 60 days' (6 July—4 Sept.) interest would be made at the current rate—say 5 per cent. (then by 146).

$$\frac{\overset{100}{\pounds 500}}{1} \times \frac{\cancel{5}}{1} \times \frac{\overset{3}{\cancel{60}}}{\underset{73}{\cancel{365}}} \times \frac{1}{\underset{\cancel{20}}{\cancel{100}}} = \frac{300}{73} = \pounds 4\ 2s.\ 2d. + = \left.\begin{matrix}\text{Int. for}\\ \text{60 dys.}\end{matrix}\right\}$$

hence £4 2*s.* 2*d.* would be charged as discount, and £495 17*s.* 10*d.* would be paid as the present value of the Bill.

Ex. XCIV. Find the discount on the following bills:

				Drawn.	Discounted.
1.	£256	10*s.*	6*d.*	March 5 at 8 months.	Aug. 27 at 5 per cent.
2.	£530	15*s.*	0*d.*	Jan. 3 at 5 months.	Jan. 11 at 4 per cent.
3.	£357	17*s.*	6*d.*	Feb. 18 at 3 months.	Mar. 9 at 5 per cent.

				Drawn.	Discounted.
4.	£1000	15*s.*	6*d.*	April 7 at 6 months.	June 17 at 2½ per cent.
5.	£565	10*s.*	6*d.*	March 4 at 4 months.	April 25 at 4 per cent.
6.	£385	15*s.*	0*d.*	Dec. 8 at 9 months.	Feb. 4 at 3 per cent.
7.	£1120	0*s.*	0*d.*	June 3 at 3 months.	June 25 at 4 per cent.
8.	£350	16*s.*	0*d.*	May 2 at 8 months.	Aug. 12 at 5 per cent.

154. The following examples, in which the discount is calculated in the manner indicated in (152), are of no practical use, but are given simply as exercises in Rule of Three.

Find the present value of £250 due 2¼ yrs. hence, at 5 p. c. per ann.

£250 is money due 2¼ yrs. hence, and to find its present value—we must have some other money, due 2¼ yrs. hence, of which we know the present value. Pres^t mon^y, £100, with its 2¼ yrs. int. add^d, will be £111¼ in 2¼ yrs. time, then ←

$$\pounds 111\tfrac{1}{4}\,\substack{(2\frac{1}{4}\text{ yrs.}\\ \text{hence})} : \pounds 250\,\substack{(2\frac{1}{4}\text{ yrs.}\\ \text{hence})} :: \pounds 100\,\substack{(\text{present}\\ \text{money})} : \substack{(\textit{present}\\ \textit{money?})}$$

Ex. XCV.

1. Find the present value of £350, due at the end of 4 years, at 3½ per cent.

2. What is the present value of £850, due 3 years hence, at 3¼ per cent.?

3. Find the present value of £650 15*s.*, due at the end of 3½ years, at 5 per cent.

4. Find the discount on £560 17*s.* 6*d.*, due 2½ years hence, at 4 per cent.

155. For Insurance, Life Assurance, Commission, and Brokerage, payments are made at a certain percentage, and the calculations are made by Rule of Three.

156. Insurance is an annual payment, at so much for every £100 worth of property, which is made secure against loss by fire, &c.

157. Life Assurance is an annual payment of so much for every £100 of the sum of money to be paid at the death of the person insured. The rate of assurance varies according to the age at which life assurance is commenced.

158. Commission is an agent's charge for buy^{g} or sell^{g} goods at so much for every £100 worth bought or sold.

159. Brokerage is a percentage charged for carrying out monetary transactions, as buying and selling stocks, shares, &c.

What will be the money paid for insuring a vessel and cargo, worth £125,000, at 2¼ per cent.? 141.

$$£100 : £125,000 :: £2\tfrac{1}{4} \;\Big|\; \frac{£9}{4} \times \frac{1250\cancel{00}}{1} \times \frac{1}{1\cancel{00}} = \frac{£11250}{4} = £2812\ 10s.$$

What is the annual premium for insuring £1000 at death, the rate being in this particular case at £3 7*s.* 6*d.* for every £100 insured? 141.

$$£100 : £1000 :: £3\tfrac{3}{8} \;\Big|\; \frac{£27}{8} \times \frac{10\cancel{00}}{1} \times \frac{1}{1\cancel{00}} = \frac{£270}{8} = £33\ 15s.$$

Ex. XCVI.

1. What will be the cash payment of the account £6 10*s.*, discount being allowed at 1*d.* on every 1*s.*?

2. What will be the ready money payment for 20 tons of coal at 25*s.* per ton, discount being allowed at 5 per cent.?

3. What will be cost of insuring a vessel worth £25,000, at 2¼ per cent.?

4 Find the annual payment for insuring from fire a house and furniture worth £9500, at 1*s.* 6*d.* per cent.

5. What would be the cash payment of an account of £157 17*s.* 6*d.*, discount being deducted at the rate of 2½ per cent.?

6. What is the commission on 1275 guineas, at 3⅛ per cent.?

7. What is the brokerage upon a money transaction of £1255 10*s.*, at £⅛ per cent.?

8. What will be the cost of insuring against fire, property valued at £1500 15*s.*, at 1*s.* 6*d.* per cent.?

9. What will be the premium on a life policy for £1500, at £3 6*s.* 8*d.* per cent.?

STOCKS.

160. Stock is the name given to money lent to the Government on the condition that so much interest shall be paid yearly for every £100 lent, but that the Government shall not be under obligation to pay back the money advanced as a loan.

161. Under such circumstances every £100 lent ceases to be regarded as money, and is spoken of as £100 worth of stock or £100 stock, and the ownership of such stock gives simply the claim for so much yearly interest.

162. The buying and selling of stock are the transfer by sale from one person to another of the claim on the Government for interest as agreed upon, at the time of the loan.

163. The *money-equivalent* of every £100 stock varies, as there may be or may not be opportunities of using money profitably in speculations or in mercantile pursuits, and such an expression as "in the 3 per cents., at 85," means that the cent. (£100 stock) gets £3 interest, and that £85 money is the market value of that stock, that is, that a buyer of stock will give £85 to have a claim on the Government for £3, considering £3 a fair interest for £85.

164. From such expression "the 3 per cents. at £85,"
int. stock money

six sets of proportionals can be written down.

3 per cents., at 85.

(*a*) Buy^{s} stk.	(*a*) £85 money : any other money : : £100 stock : (*stock?*)
(*b*) Sell^{s} stk.	(*b*) £100 stock : any other stock : : £85 money : (*money?*)

3 per cents. at 85.

(*c*) Find^{s} int. on stk.	(*c*) £100 stock : any other stock : : £3 int. : (*int.?*)
(*d*) Find^{s} stk. wh. gets int.	(*d*) £3 int. : any other int. : : £100 stk. : (*stock?*)

Inasmuch as money buys stock and stock gets interest,
then money gets interest.

3 per cents., at 85.

(*e*) Find^{s} int on money.	(*e*) £85 money : any other money : : £3 int. : (*int.?*)
(*f*) Find^{s} mny. wh. gets int.	(*f*) £3 int. : any other int. : : £85 mny. : (*mon?*)

How much stock can be bought for £4000, in the 3 per cents. at 90.

(*a*) £90 money : £4000 money : : £100 stock : (*stock?*)

How much money will be realized by selling £3000 stock in the 4 per cents., at 95?

(*b*) £100 stock : £3000 stock : : £95 money : (*money?*)

If £4500 be invested in the 3½ per cents., at £95, what annual income will be obtained?

(*e*) £95 money : £4500 money : : £3½ int. : (*int.?*)

The 3½ per cents. are at 96; how much money must be invested to give an income of £150?

(*f*) £3½ income : £150 income : : £96 money : (*money?*)

by the Unitary Method. pp. 85, 86.	£3½ income belongs to £96 mon^{y} invest^{d}.
	£1 " " $\frac{£96}{3\frac{1}{2}}$ " "
	£150 " " $\frac{£96}{3\frac{1}{2}} \times 150$ "

Ex. XCVII.

1. The 3 per cents. being at 95, what money will be required for the purchase of £5000 stock? (*b*).

2. If £1250 be spent in the purchase of 4 per cent. stock at 90, what annual interest will be obtained? (*e*).

3. A person has left to him by will £10,000 stock in the 3½ per cents. at 85; what income will he derive from it? (*c.*)

4. A person sells £6500 stock in the 4 per cents. at 82, what money will he receive from the buyer? (*b.*)

5. What is the price of stock, when £3000 money will buy £5000 stock in the 3 per cents.? (*b.*)

6. How much stock must be held in the 4 per cents. at 85, to get an income of £170? (*d.*)

7. How much money must be invested in the 4 per cents. at 85, to get an income of £170? (*f.*)

8. If I buy £6000 stock in the 3½ per cents. at 70, how much money will the stock cost me? (*b.*)

9. If I hold £8500 stock in the 3 per cents. at 96, how much money could I realise by selling? (*b.*)

10. How much stock must be held in the 4 per cents. at 90, to receive a yearly interest of £200? (*d.*)

11. What money must be invested in the 3 per cents. at 75, to get a yearly income of £50? (*f.*)

12. When the 4 per cents. are at 90, what money invested in them will bring an income of £144? (*f.*)

EXAMPLES ON TRANSFER OF STOCK.

165. It is to be remembered that the transfer of stock does not signify the exchange of one kind of stock for another. Stock held, must be sold, and other stock must then be purchased with the money.

166. It should also be noted that from such an expression as "3 per cents., at 85," *interest* can be *found on stock* (164*c*), and *interest* can be *found on money* (164*e*).

If a person were to transfer £25,000 stock from the $3\frac{1}{2}$ per cents. at 90, to the 3 per cents. at 95, what stock would he hold in the latter? (164 *b*, *a*.)

Sell^{g} stock at 90.

(*b*) £100 stock : £25000 stock : : £90 money : (*money?*)

$$\text{Money } \frac{£90 \times 25000}{100} = £22{,}500 \text{ money.}$$

Buy^{g} stock at 95.

(*a*) £95 money : £22500 money : : £100 stock : (*stock?*)

$$\text{Stock } \frac{£100 \times \overset{4500}{\cancel{22500}}}{\underset{19}{\cancel{95}}} = \frac{450000}{19} = £23684\tfrac{4}{19} \text{ stock.}$$

If a person were to transfer £35000 stock from the 3 p. c. at 85. to the 4 p. c. at 90, what would be the difference in his income? (164 *c*, *b*, *e*.)

Find^{g} inco. on stk. in 3 p. c.

(*c*) £100 stock : £35000 stock : : £3 income : (*income?*)

$$\text{Income } \frac{£3 \times 35000}{100} = £1050 \text{ income.}$$

Sell^{g} £35,000 stk. at 85.

(*b*) £100 stock : £35000 stock : : £85 money : (*money?*)

$$\text{Money } \frac{£85 \times 35000}{100} = £29{,}750 \text{ money.}$$

Find^{g} int. on. £29750 mon^{y} in 4 p. c. at 90.

(*e*) £90 money : £29750 money : : £4 income : (*income?*)

$$\text{Income } \frac{£4 \times 2975\cancel{0}}{9\cancel{0}} = \frac{11900}{9} = £1322\ 5s.\ 6\tfrac{2}{3}d. \text{ income.}$$

$$£1322\ 5s.\ 6\tfrac{2}{3}d. - £1050 = £272\ 5s.\ 6\tfrac{2}{3}d. \text{ diff. of inc.}$$

13. What would be the amount of stock held in the 4 per cents., at 75, if a person were to transfer to that investment £15,000 stock, in the 3½ per cents. at 90? 164 (*b*), (*a*).

14. If a person transfer £10000 stock from the 4 per cents. at 96, to the 3 per cents. at 95, what is the difference in his income? 164 (*c*), (*b*), (*e*).

15. What would be the difference in income between £1530 invested in the 4 per cents. at 90, and the same money in the 3 per cents. at 85? 164 (*e*).

16. A person invests £15000 in the 3½ per. cents. at 96, and on their rising to 98 sells out, how much money would he clear by the transaction? 164 (*a*), (*b*).

17. In which is it most advantageous to invest, in the 3 per cents. at 89, or the 4 per cents. at 95? 164 (*e*).

18. A person invests £16200 in the 3 per cents., at 90, and on their rising to 91 transfers to the 3½ per cents. at 95, what is the difference in his income? 164 (*c*), (*b*), (*e*).

19. Which is the best investment, the 4 per cents. at 96, or the 3½ per cents. at 85? 164 (*e*).

PROFIT AND LOSS.

167. The rate of gain or loss is usually expressed at so much per cent.

168. The *money used* in the purchase of goods, &c., is the money which *gains or loses*, and the gain or loss per cent. is the gain or loss by the use of £100 in buying. Thus, if £100 worth of iron were bought, and then sold for £105, the £100 called the *cost* would have gained £5, and *the rate of gain* would be 5 *per cent.*

FORMS OF STATING.

				Material.		*Answer.*
cost	:	cost	::	sell^{g} pr.	:	(*sell^{g} pr.?*)
sell^{g} pr.	:	sell^{g} pr.	::	cost	:	(*cost?*)
cost	:	cost	::	gain	:	(*gain?*)
cost	:	cost	::	loss	:	(*loss?*)

169. The expression "to gain 5 per cent." gives two connected terms —£100 *cost* and £105 *selling price.*

(*a*) to find sell^{g} pr.	(*a*) £100 cost	:	any cost	::	£105 sell^{g} pr.	: (*sell^{g} pr.?*)
(*b*) to find cost	(*b*) £105 sell^{g} pr.	:	any sell^{g} pr.	::	£100 cost	: (*cost?*)

The express^n to lose 5 p. c. gives £$\overleftarrow{\underrightarrow{\text{100 cost, £95}}}$ sell^s pr.

(*c*) to find sell^s pr.	(*c*) £100 cost : any cost :: £95 sell^s pr. : (*sell^g pr.* ?)
(*d*) to find cost.	(*d*) £95 sell^s pr. : any sell^s pr. :: £100 cost : (*cost* ?)

Since the mon^y used (cost) is the mon^y which gains or loses,

(*e*) to find gain p. c.	(*e*) cost : £100 cost :: gain : (*gain per cent.* ?)
(*f*) to find loss p. c.	(*f*) cost : £100 cost :: loss : (*loss per cent.* ?)

the first and third terms being connected, *i.e.* the first being ***a certain** cost*, and the third ***its** gain* (*e*), ***its** loss* (*f*).

If iron be sold at £3 1*s.* 6*d.* per cwt., at a loss of 5 per cent., what was its cost? (*d*)

"loss of 5 per cent." gives £$\overleftarrow{\text{100 cost, £9}}$5 sell^g pr.

(*d*) £95 sell^s pr. : £3 1*s.* 6*d.* sell^g pr. :: £100 cost : (*cost* ?)

by the Unitary Method, pp. 85, 86.	£95 sell^s pr. belongs to £100 cost. £1 " " $\frac{£100}{95}$ " £$3\frac{3}{40}$ " " $\frac{£100}{95} \times 3\frac{3}{40}$.

(*x*) sell^s price − cost = gain. (*y*) cost − sell^s price = loss. (*z*) such sums of money must belong to same weight, same quantity, &c., before subtraction.

170. In some questions, it is required to find cost corresponding to selling price or selling price to cost, and *also to change by* Multiplication, Division, or Rule of Three *from one quantity to another.*

(*a*) If 18 yds. be bought for 36*s.*, and sold at a gain of 5 per cent., what is the selling price of 1 E. ell.? (*a.*)

To change from 18 yds. to 1 E. ell.	$\begin{array}{cccc} 18\text{ yds.} & : 1\text{ E. ell} & :: \text{——} & : (\text{——}?) \\ \underline{4} & \underline{5} & & \\ 72\text{ qrs.} & 5\text{ qrs.} & & \end{array}$

Hence, to change quantity $\times \frac{5}{72}$.

(*a*) £100 cost : 36*s.* cost :: £105 sell^s pr. : (*selling pr.* ?)

$$\frac{£105}{1} \times \frac{36}{1} \times \frac{1}{2000} \times \frac{5}{72} = \text{sell}^s \text{ pr. of 1 E. ell.}$$

(*b*) If 3 per cent. be lost by selling silk at 15*s*. 6*d*. a yd., at what price per yard must it be sold to gain 3 per cent.?

£100 worth at the first price is sold for £97.
" sec^{d} " " £103.

£97 first sell^{g}	:	15*s*. 6*d*. first sell^{g}	::	£103 second sell^{g}	:	*second selling*
pr. of		pr. of		pr. of		*pr. of*
£100		1 yd.		£100		1 *yd.*
worth.				worth.		

Ex. XCVIII.

1. What will be the selling price of silk, which cost 9*s*. 6*d*. per yd. to gain 5 per cent.? (*a*).

2. If silk sold at 12*s*. per yd. bring a gain of 10 per cent.; what was the cost per yd.? (*b*).

3. By selling at £1 10*s*. per cwt., there is a loss of 8 per cent.; what was the cost? (*d*).

4. If the cost of a yd. be 7*s*. 6*d*., and the selling price 9*s*.; what is the gain per cent.? (*e*).

5. If sugar be bought at £1 15*s*. 9*d*. per cwt., and sold at £1 15*s*. per cwt.; what is the loss per cent.? (*y*), (*f*).

6. If 144 yds. are bought at 17*s*. 6*d*. per yd., and sold for £144; what is the gain per cent.? (*x*), (*z*), (*e*).

7. If cheese be sold for £3 6*s*. 6*d*. per cwt., at a loss of 12 per cent.; what was the cost per cwt., and what per lb.? (*b*) 170 (*a*).

8. If 60 ells cost £100, how must the selling price per yd. be fixed to gain 15 per cent.? (*a*) 170*a*.

9. If 17 cwt. of cheese be sold for £68 10*s*. at a gain of 5 per cent.; what was the cost per lb.? (*b*) 170 (*a*).

10. Bought 18 yds. at 4*s*. 6*d*. per yd., 20 yds. at 3*s*. 6*d*., and 32 yds. at 5*s*. 9*d*.; for how much must the whole be sold to gain 2½ per cent.? (*a*)

11. At what price per lb. would 1 cwt. of sugar, which cost £2 16*s*. per cwt., be sold to lose 2½ per cent.? (*c*) 170 (*a*).

12. What will be the loss per cent., if sugar bought for £50 12*s*. 6*d*. be sold for 40 guineas? (*y*), (*f*).

13. If 16 tons be bought for £18 17*s*. 6*d*. and sold at a loss of 15 per cent.; what is the selling price of 1 cwt.? (*c*) 170 (*a*).

14. If by selling at 6*s*. per yd., there is a gain of 5 per cent., how much per cent. will be gained by selling at 7*s*. 6*d*. per yd.? (*b*), (*x*), (*e*).

15. If cheese cost £3 14*s*. 8*d*. per cwt., how must it be sold per lb. to gain 5 per cent., and also to lose 5 per cent.? (*a*) 170 (*a*), (*c*), 170 (*a*).

16. In gaining 1½*d*. on every 1*s*., what is the gain per cent.? (*e*)

17. If 3 per cent. be gained by selling at 2*s*. 6*d*. per yd., at what price will 8 per cent. be gained? 170*b*.

18. What selling price per cwt. will bring a loss of $4\frac{1}{8}$ per cent. on 15 cwt. which cost £7 16*s*. 6*d*. per cwt.? (*c*), 170 (*a*).

DIVISION INTO PROPORTIONAL PARTS.

171. *To divide a given number or quantity into parts, which shall have to each other particular ratios*, from the numbers or fractions given as the representatives of the several parts required, find a representative of the whole given number or quantity, and then work out by Rule of Three.

(*a*) Divide £100 among two persons, giving 3 shares to one, and 7 shares to another.

Here 10 shares represent the whole, or £100

10 shares : 3 shares :: £100.
10 shares : 7 shares :: £100.
or find 1 share.
10 shares : 1 share :: £100.
other shares found by multiplication.

(*b*) Divide £500 among A B and C, giving A half as much as B, and C as much as A and B together.

Let 1 represent B's share
then 2 represents A's „
„ 3 „ C's „
„ 6 „ the whole.

6 : 1 :: £500
other shares found by multiplication.

(*c*) A B and C provide respectively £500, £600, £700, of capital, find their respective shares of a gain of £600.

£500 + £600 + £700 = £1800

£1800 cap. : £500 cap. :: £600 gain : (*A's gain?*) &c.

(*d*) If A provide £500 capital for 6 months, B £600 for 4 months, C £1000 for 9 months; find their respective shares of a gain of £1500.

£500 for 6 mths. = £3000 for 1 mth.
£600 „ 4 „ = 2400 „ „
£1000 „ 9 „ = 9000 „ „
14400 „ „

£14400 : £3000 :: £1500 : (*A's sh.?*)
&c.

Ex. XCIX.

1. Divide 5325 into parts which shall have the ratio of 3 : 5; also of $\frac{1}{3} : \frac{1}{5}$.

2. *A.* and *B.* gain in trade £750, with a capital of £1200; of which *A.* provided £800 and *B.* 400; find *A.'s* share of the gain.

3. In the proportion of 100 parts of copper and 12 parts of tin, find how much copper there will be in 8 cwt. 2 qrs. 14 lbs.

4. Divide £100 among 3 persons, giving the second twice as much as the first, and the third as much as the first and second together.

5. If *A.* provide a capital of £500 for 13 weeks, *B.* £400 for 6 weeks, and *C.* £100 for 13 weeks, what part of a gain of £425 will belong to *C.*?

6. The rent of a field is £10 17*s.* per annum; *A.* puts in 4 horses for 12 weeks; *B.* 6 horses for 25 weeks, and *C.* 1 horse for 19 weeks; what portion of the rent should *A.* pay?

SQUARE ROOT.

172. When any abstract number is multiplied by itself, the product is called the square of that number; thus, 6 × 6 gives 36 as the square of 6; and the number multiplied by itself is called the square root of such product; thus 6 is the square root of 36.

Square Root.	Square.	Square Root.	Square.
1	1	6	36
2	4	7	49
3	9	8	64
4	16	9	81
5	25		

In the foregoing Table 4 is the sq. of 2, and 2 the sq. root of 4, &c. Written $2^2 = 4$, $\sqrt{4} = 2$, &c.

The following examples will illustrate the method of extracting the square root of a given number. The proof of such method is best shown by Algebra.

Find the square root of 190096.

Divide the given number into periods of two figures each, commencing with units;

beginning with left-hand period (19) find from Table the nearest square that can be subtracted from it, i.e. 16 of which *root is* 4. Subtract and bring down next period;

double root figure 4 = 8 and call it for the moment 80. Using 80 as a temporary trial divisor into 300 we get 3. Divide by 83 and bring down next period;

double root figures 43 = 86 and call them for the moment 860. Using 860 as a temporary trial divisor into 5196 we get 6. Divide by 866.

		square. sq. rt.
		19|00|96(436
		16
~~80~~	83	300
		249
~~860~~	866	5196
		5196

		76|38|76 (874
		64
~~160~~	167	1238
		1169
~~1740~~	1744	6976
		6976

		1|47|62|25 (1215
		1
~~20~~	22	47
		44
~~240~~	241	362
		241
~~2420~~	2425	2125
		2125

173. Integers and decimal figures must be marked off into periods separately, *starting from* the *decimal point to left* for integers *and* to *right* for decimals. Should there be an odd decimal figure, a cypher must be added to make the lowest period complete. The square root of integer periods will be integers, of decimal periods, decimals.

		1|47·62|25 (12·15
		1
~~20~~	22	47
		44
~~240~~	241	362
		241
~~2420~~	2425	12125
		12125

			·00|05|30|84|16 (·02304
			4
	~~40~~	43	130
			129
~~460~~	~~4600~~	4604	18416
			18416

174. The following examples may be noted, as illustrating, under different circumstances, the shortest methods by which the square root of a vulgar fraction may be found:—

Find the square root of $\frac{81}{121}$, $\frac{140}{225}$, $\frac{3}{5}$, $\frac{4}{7}$.

$$\sqrt{\frac{81}{121}}=\frac{\sqrt{81}}{\sqrt{121}}=\frac{9}{11}. \qquad \sqrt{\frac{140}{225}}=\frac{\sqrt{140}}{\sqrt{225}}=\frac{\sqrt{140}}{15}=\frac{11\cdot82, \&c.}{15}=\cdot78, \&c.$$

$$\sqrt{\frac{3}{5}}=\sqrt{\cdot6}=\cdot774, \&c. \qquad \sqrt{\frac{4}{7}}=\frac{\sqrt{4\times7}}{\sqrt{7\times7}}=\frac{\sqrt{28}}{\sqrt{49}}=\frac{\sqrt{28}}{7}=\frac{5\cdot28,\&c.}{7}=\cdot75, \&c.$$

Ex. C.

Find the square root of—

1. 7513081
2. 9030025
3. 39·0625
4. 9564·84
5. ·000089415936
6. ·0000903005
7. $7\frac{9}{16}$
8. $1\frac{56}{169}$
9. $464\frac{52}{81}$
10. $20\frac{44}{49}$

CUBE ROOT.

175. When the square of a number is multiplied by that number, the result is called the cube; thus $8^2=64\times8=512$, which is the cube of 8, and 8 is said to be the cube root of 512. Written $8^3=512$, $\sqrt[3]{512}=8$.

Cube root.	Cube.	Cube root.	Cube.
1	1	6	216
2	8	7	343
3	27	8	512
4	64	9	729
5	125		

The following examples will illustrate the method of finding the cube root of a given number; the proof of such method is best shown by algebra:—

Find the cube root of 94818816.

Mark off into sets of three figures each; in the case of integers, commence with units and mark off to the left; of decimals, begin with tenths and mark off to the right. The lowest period of decimals must be made complete, if necessary, by adding cyphers.

Find the highest cube (64) that can be subtracted from the left hand period (94); subtract and bring down the next period 818; place root figure of 64, *i.e.* 4 in answer.

Multiply root figure (4) by 3 and place on left of remainder. Multiply the number thus obtained (12) by last root figure (4) = 48 and add two cyphers. 4800 as a temporary trial divisor gives 5; place 5 at side of 12 making 125, which multiply by 5 and adding to trial divisor, make the divisor used, subtract and bring down next period.

Multiply root figures (45) by 3 = 135, add square of last root figure, i.e. $5^2 = 25$ to the last divisor and the number immediately above,

$\left.\begin{array}{r}625\\5425\\25\end{array}\right\} = 6075,\dagger$ add two cyphers.

507500 as a trial divisor gives 6; place 6 to right of 135 making 1356, which multiply by 6 and add to trial divisor.

		94|818|816 (456
		64
		—
125	4800	30 818
	625	
	5425	27 125
1356	†607500	36 938 16
	8136	
	615636	36 938 16
		....

Find the cube root of 40177390625.

			40|177|390|625 (3425
			27
	94	2700	13 177
376		376	
3076		3076	12 304
16 = 4^2	1022	*346800	873 390
3468*		2044	
2044		348844	697 688
348844	10265	†35089200	175 702 625
4 = 2^2		51325	
350892†		35140525	175 702 625

Ex. CI.

Find the cube root of

1. 2197.
2. 1953125.
3. 405224.
4. 529475129.
5. 21717639.
6. ·000000003375.
7. 84451·88603.
8. $\frac{343}{729}$.
9. $31\frac{15}{343}$.
10. $69\frac{352}{729}$.

MISCELLANEOUS EXAMPLES (LXXXIX—CII.)

1. Find the simple interest on £1215 17*s.* 6*d.* for 3½ yrs. at 4⅛ per cent. per ann. 145.

2. Find the simple interest on £500 10*s.* 6*d.* for 3 yrs. 5 m. at 2½ per cent. per ann. 146.

3. Find the simple interest on £1515 5*s.* from 7 May 1882, to 6 June 1883, at 3 per cent. per ann. 146.

4. What is the compound interest on £500 12*s.* 6*d.* for 2 yrs. at 5 per cent.? 147, 148.

5. In what time will £125 10*s.* amount to £251, at 5 per cent. per ann.? 149, CASE I.

6. What sum of money will amount to £525 14*s.* in 3 yrs., at 4 per cent.? 149, CASE II.

7. At what rate per cent. will £500 amount to £525 in 3 yrs.? 149, CASE III.

8. Find the amount of £500 15*s.* for 3½ yrs. at 2¼ per cent. 144, 145.

9. Find the discount on an account which amounts to £55 6*s.* 6*d.* at 2½ per cent. 151, 141.

10. Find the present value of £575 due 3 yrs. hence, at 5 per cent. 154.

11. What will be the money paid for insuring a ship and cargo, worth £15000, at 3½ per cent.? 141.

12. What is the commission on a purchase of goods of the value of £125 15*s.* at 2¼ per cent.? 141.

13. How much stock can be bought for £1225 10*s.* in the 3½ per cents., at 80? 164*a*.

14. What income will £1225 10*s.* bring, if invested in the 3½ per cents., at 80? 164*e*.

15. When the 3½ per cents. are at 95, what money must be invested to bring an income of £250? 164*f*.

16. What money will be required to purchase £10,500 stock in the 4 per cents. at 98? 164 *b*.

17. If a person invests £5000 in the 3 per cents., at 75, and on their rising to 80, transfer to the 4 per cents. at 90, what will be the difference in his income? 164 *e*, *a*, *b*, *e*.

18. What will be the gain per cent., if goods bought at 7*s*. 6*d*. per yd. be sold at 9*s*. per yd.? 169 *x*, *e*.

19. How must iron bought at £3 7*s*. 6*d*. per cwt., be sold to gain 1½ per cent.? 169 *a*.

20. If 18 yds. be bought for £35 15*s*., and sold at a gain of 3½ per cent., what is the selling price of 1 yd.? 169 *a*, 170.

21. What selling price per yd. will make a loss at the rate of 1½ per cent., when 16 yds. cost 17*s*. 6*d*.? 169 *c*, 170.

22. Divide £500 among three persons in the ratio of 1, 2, 3. 171.

23. Find the square root of ⅓ and of ·5. 173, 174.

24. In what time will £500 amount to £750 15*s*. at 4 per cent. per annum? 149, CASE I.

25. Find the compound interest on £625 17*s*. 6*d*. for 2 yrs. at 5 per cent. 147, 148.

26. What amount of stock in the 3½ per cents. at 90, would yield an income of £255 15*s*.? 164 *d*.

27. If a person transfer £5000 stock from the 3½ per cents. at 85, to the 4 per cents. at 95, what would be the difference in his income? 164 *c b e*.

28. In which is it most advantageous to invest, the 4 per cents. at 80, or the 3½ per cents. at 75? 164 *e*.

29. If 14 c. 1 qr. 7 lbs. be bought for £17 17*s*. and sold at a gain of 10 per cent., what is the selling price of 3 qrs. 14 lbs.? 169 *a*, 170.

30. If three persons enter into partnership and provide capital as follows: *A*, £600 for 3½ months, *B*, £500 for 2¾ months, and *C*, £1000 for a year; what share of a profit of £600 will belong to *C*? 171 *d*.

31. Find the square root of $5\frac{1}{25}$, $\cdot\dot{7}$ and of $4\frac{1}{3}$. 174, 173.

32. Find the discount on a bill for £725 17*s*. 6*d*. drawn on 3 April 1875, at 6 months, and discounted on 13 May, at 4½ per cent. 153 *ex*.

33. Find the amount of £1216 18*s*. 6*d*. from 1 March 1883, to 17 June 1884, at 5 per cent. per annum. 144, 146.

34. In what time will £1000 become £1500 at 4½ per cent. per annum? 149, CASE I.

35. If tea be bought at 5*s*. 6*d*. per lb., and sold at 6*s*. 8*d*. per lb., what is the gain per cent.? 169 *x*, *e*.

36. If £12000 stock be transferred from the 4 per cents. at 70½, to the 3½ per cents. at 68, how much stock will be held after the transfer, and what income will be received? 164 *b*, *a*, *c*.

37. How much per English ell was given for 17 yds. 3 qrs. of cloth, which, sold at a gain of 15 per cent., realized £8 8*s*.? 169 *b*, 170 *a*.

38. At what rate per cent. will £1 become a guinea in 9 months? 149, Case III.

39. Find the discount allowed for immediate payment at $2\frac{1}{3}$ per cent., on an account of £17 17*s.* 6*d.* 141. ,

40. What will be charged for the insurance of a ship and cargo, worth £17000 17*s.* 6*d.* at $3\frac{1}{6}$ per cent.? 141.

41. How much stock must be held in the $3\frac{1}{2}$ per cents. at 95, to give an income of £500 per annum? 164 *d.*

42. By selling at £1 15*s.* per cwt., there is a loss of 18 per cent.; what was the prime cost of 1 lb.? 169 *d*, 170.

43. If 3 cwt. 1 qr. 18 lbs. be bought for £16 15*s.* 6*d.*, and sold for £18 18*s.*, what is the gain per lb., and what is the gain per cent.? 169 (*x*), (*e*).

44. Find the square root of 1725·172, and of 17 $\frac{1}{144}$. 173, 174.

45. Find the cube root of ·000000027. 175.

46. Find the discount on a bill drawn for £175 15*s.* 6*d.* on 1 January 1883 at 4 months, and discounted at 4 per cent. on 20 February. 153.

47. What is the commission on 1000 guineas, at £$\frac{1}{8}$ per cent.? 141.

48. Which investment gives the higher rate of interest, the $3\frac{1}{2}$ per cents. at 91, or the 4 per cents. at $93\frac{7}{8}$? 164 *e.*

49. What income would be obtained in the 4 per cents. at 75, if a person were to transfer to that investment £15000 stock in the $3\frac{1}{2}$ per cents. at $83\frac{7}{8}$? 164 *b e.*

50. If 4 per cent. be gained by selling at £3 3*s.* per cwt., at what price per lb. would 12 per cent. be gained? 170 *b.*

ANSWERS.

Ex. I, p. 2.

1. Five millions, seven hundred and twenty-four thousand, one hundred and fifty-eight.

2. Thirty-nine millions, two hundred and fifty-four thousand, three hundred and seventeen.

3. Eight hundred and forty-nine millions, six hundred and thirty-one thousand, two hundred and eighty-seven.

4. Three thousand seven hundred and forty-two millions, one hundred and ninety-five thousand, one hundred and sixty-four.

5. Seven thousand two hundred and eighty-five millions, four hundred and nineteen thousand, six hundred and thirty-two.

6. Eighteen thousand seven hundred and forty-two millions, one hundred and fifty-eight thousand, seven hundred and sixty-three.

7. Two hundred and sixteen millions, four hundred and twenty-five thousand, three hundred and twenty-eight.

8. Three hundred and fifty-two millions, one hundred and eighty-seven thousand, six hundred and fifty-four.

9. Three hundred and seventy-two millions, five hundred and sixteen thousand, four hundred and eighty-three.

10. Four hundred and fifty-six millions, seven hundred and eighty-nine thousand, one hundred and twenty-three.

Ex. II, p. 3.

1. Thirty millions, five thousand and twenty.

2. Three hundred and five thousand and two.

3. Three millions, one thousand and twenty-five.

4. One hundred and forty-seven millions, fifteen thousand and fifteen.

5. Three thousand and sixteen millions, fifty-one thousand, one hundred and sixty.

6. One hundred and twenty-seven thousand and seven millions, seventy thousand and seven hundred.

7. Two thousand and seven millions, fifteen thousand and three.

8. Three thousand three hundred millions, three thousand and three hundred.

9. Ten thousand and seven millions, three thousand, three hundred and five.

10. Four thousand and nineteen millions, seventy thousand, seven hundred and seven.

Ex. III, p. 3.

1. 206349418284. 2. 29306242958. 3. 8800082.
4. 118000027. 5. 3407502096008. 6. 616410303097.
7. 13003007005. 8. 4040014.

Ex. IV, p. 4.

1. 1603409442. 2. 951307495. 3. 1100049991.
4. 18717. 5. 1002800. 6. 1287287286.

Ex. V, p. 5.

1. 1317213. 2. 1711463. 3. 1048631. 4. 1319848
5. 2015631. 6. 24933448. 7. 20400165. 8. 25311336.
9. 34440718. 10. 28682946. 11. 34771950. 12. 33795234.
13. 1138119. 14. 1236450. 15. 978824379. 16. 1315424217.

Ex. VI, p. 7.

1. 25352451. 2. 3371729. 3. 11983118. 4. 456169.
5. 2827749. 6. 4336609. 7. 486448. 8. 7132367.
9. 13777786. 10. 9787093. 11. 11777679. 12. 13675365.
13. 17476. 14. 729078. 15. 11562240. 16. 294999955.
17. 1785193981.

Ex. VII, p. 8.

1. 177. 2. 3101. 3. 201. 4. 5811. 5. 399. 6. 178.

Ex. VIII, p. 9.

1. 690868. 2. 1102542. 3. 942984. 4. 3690745.
5. 1289268. 6. 1654415. 7. 1457384. 8. 5158116.
9. 9356470. 10. 2798048. 11. 7725900. 12. 5704416.

Ex. IX, p. 10.

1. 8310912. 2. 14858900. 3. 33061275. 4. 25096608.
5. 81342624. 6. 56523501. 7. 52290997. 8. 35447808.
9. 126222192. 10. 20629728. 11. 23897328. 12. 37366483.

Ex. X, p. 11.

1. 639161136. 2. 88232424. 3. 356524220. 4. 341765217.
5. 121510776. 6. 237985968. 7. 395684916. 8. 313282620.
9. 262050506. 10. 342873363. 11. 11735379306. 12. 17721259201.

Ex. XI, p. 11.

1. 4086007200. 2. 1507772200. 3. 12635581000.
4. 2569199094. 5. 9554335560. 6. 1064395200.
7. 1742855535. 8. 5287316320. 9. 2297388990.

Ex. XII, p. 13.

1. 307617. 2. 173105, rem. 2. 3. 205886, rem. 2.
4. 115298, rem. 3. 5. 65709. 6. 77679, rem. 3.
7. 53578, rem. 1. 8. 44178, rem. 7. 9. 22579, rem. 9.
10. 41488, rem. 9. 11. 66483, rem. 6. 12. 15877, rem. 3.

Ex. XIII, p. 13.

1. $240 = 2 \times 3 \times 5 \times 8$; $112 = 4 \times 7 \times 4$; $1728 = 12 \times 12 \times 12$; $1512 = 3 \times 9 \times 8 \times 7$; $720 = 2 \times 3 \times 4 \times 5 \times 6$; $1760 = 8 \times 2 \times 11 \times 10$.

2. $5 \times 5 \times 3$; $9 \times 4 \times 12$; $5 \times 11 \times 11$; $12 \times 12 \times 12$; $3 \times 9 \times 10$; $4 \times 8 \times 9$; $5 \times 3 \times 7$; $7 \times 7 \times 7$; $8 \times 8 \times 10$; $8 \times 2 \times 11 \times 10$.

3. $8 \times 11 \times 2$; $10 \times 10 \times 10$; $5 \times 5 \times 7$; $8 \times 7 \times 2$; $5 \times 11 \times 10$.

Ex. XIV, p. 16.

1. 131530, rem. 6. 2. 85608. 3. 124220, rem. 10.
4. 78445, rem. 58. 5. 57522, rem. 51. 6. 1766071, rem. 12.
7. 36008, rem. 12. 8. 25422, rem. 44. 9. 55328, rem. 28.
10. 69828, rem. 5. 11. 57458, rem. 20. 12. 62404, rem. 23.

1. 422782, rem. 44. 2. 32221, rem. 1044. 3. 37847, rem. 101.
4. 381858, rem. 80. 5. 782998, rem. 110. 6. 4423, rem. 953.
7. 13977, rem. 873. 8. 142878, rem. 59. 9. 80410, rem. 112.

Ex. XV, p. 16.

1. 2068255, rem. 8. 2. 797185, rem. 10. 3. 959107, rem. 15.
4. 629904, rem. 43. 5. 545059, rem. 42. 6. 371252, rem. 29.
7. 122042, rem. 194. 8. 203278, rem. 162. 9. 77217, rem. 297.
10. 69377, rem. 581. 11. 247891449, rem. 707. 12. 172240416, rem. 529.

Ex. XVI, A, p. 17.

1. 144. 2. 42. 3. 3467.
4. 21316. 5. 576. 6. 34.

Ex. XVI, B, p. 17.

1. 144. 2. 379512. 3. 96822.
4. 400. 5. 379543. 6. 83329.

Miscellaneous Examples, pp. 18, 19.

1. Three hundred and ninety-eight thousand, four hundred and seventy-six.

2. Forty thousand and seven.

3. 166284. 4. 10049. 5. 232583. 6. 496566.
7. 409500938. 8. 409500938. 9. 4409701, rem. 24.
10. 4409701, rem. 24.

11. Two hundred and eighteen millions, five hundred and thirteen thousand, one hundred and twenty-seven.

12. Fifty millions, fifty thousand and five hundred.

13. 6672584. 14. 3004004. 15. 84652. 16. 24691.
17. 60679728. 18. 60679728. 19. 136, rem. 119.
20. 136, rem. 119.

21. One hundred and twenty-three millions, four hundred and fifty-six thousand, seven hundred and eighty-nine.

22. Twenty millions, one thousand and one.

23. 1012000. 24. 1644734. 25. 10759, rem. 14.
26. 85576, rem. 9. 27. 55117, rem. 16. 28. 102866, rem. 52.
29. 258417, rem. 18. 30. 148945, rem. 19.

31. Fifteen billions, one hundred and seventy-six thousand two hundred and thirteen millions, five hundred and twelve thousand, one hundred and sixteen.

32. One billion, one thousand one hundred millions, ten thousand one hundred.

33. 502084006309. 34. 1213212.

35. Two millions, two hundred and eighty-four thousand, five hundred and sixty-one.

36. Nine hundred and ninety-one thousand, six hundred and twenty-eight.

37. 22954451, rem. 1. 38. 239170, rem. 833.
39. 1572814, rem. 9. 40. 4697, rem. 25745.
41. 4283. 42. 2337. 43. 2767. 44. 348.
45. 738. 46. 12. 47. 9912672. 48. 34.
49. 713453. 50. 3940.

Ex. XVII, p. 20.

1. 401280 far.; £164 18*s.* 5¾*d.* 2. 183120*d.*; £340 12*s.* 10*d.*
3. 4524*d.*; 15586 far. 4. 13365 halfpence; 10032 halfpence.
5. 1941 sixpences; 1180*s.* or 14160*d.*

6. 7714380 halfpence ; 433 gui. 9*s.* 4*d.*
7. 24434 florins and 3½*d.* over ; 39094 half crowns and 1*s.* 7*d.* over.
8. 1504 gui. 16*s.* 7¼*d.* ; 100429665*s.* or £5021483 5*s.*
9. 115863600 far. ; 507 crs. and 1*s.* 5*d.* over.

Ex. XVIII, p. 21.

1. 1904 lb. ; 209664 lb.   2. 4608 drs. ; 3 cwt. 3 qrs. 5 lb. 11 oz. 14 drs.
3. 4144 oz. ; 10584 lb.
4. 132 tons 19 cwt. 0 qrs. 25 lb. 1 oz. ; 437296 drs.
5. 1722 qrs. 9 lb. 4 oz. 14 drs. ; 2845 tons 13 cwt. 1 qr.
6. 113488 oz. ; 813952 drs.
7. 6144 cwt. 2 qrs. 4 lb. 3 oz. ; 21 tons 12 cwt. 2 qrs. 1 lb. 7 oz. 7 drs.
8. 264304 oz. ; 994032 drs.
9. 6889 cwt. 1 qr. 8 lb. 14 oz. ; 31545 lb.

Ex. XIX, p. 21.

1. 855 lb.   2. 999 lb.   3. 1054 lb.   4. 747 lb.
5. 1945 lb.   6. 2059 lb.   7. 12943 lb.   8. 3117 lb.

Ex. XX, p. 21.

1. 3717 grs. ; 19207 grs.
2. 253 lb. 8 oz. 8 dwt. 13 grs. ; 239067 oz. 16 dwt.
3. 1926 grs. ; 8964 lb. 0 oz. 6 dwt. 2 grs.
4. 214 lb. 5 oz. 5 dwt. 8 grs. ; 5073 lb. 6 oz. 11 dwt.

Ex. XXI, p. 22.

1. 69640 grs. ; 8183 lb. 9 oz. 2 drs. 1 scr. 6 grs.
2. 2348 lb. 3 oz. 4 drs. 1 scr. 4 grs. ; 17301 grs.
3. 1978 grs. ; 4936 lb. 4 oz. 7 drs.

Ex. XXII, p. 22.

1. 262417 in. ; 818 m. 2 f. 10 p. 2 yds. 0 ft. 7 in.
2. 33444 in. ; 45229 p. 3 yds. 1 ft. 6 in.
3. 252 ft. 1 in. ; 25921 p. 2 yds. 2 ft. 2 in.
4. 665292 in. ; 976 m. 4 f. 2 p. 3 yds.
5. 144723 in. ; 3845 ft.
6. 157 m. 6 f. 24 p. 5 yds. 2 ft. 4. in. ; 19580 yds.
7. 10834560 in. ; 1560 l. 1 m. 1 f. 3 p. 1 yd. 2 ft. 6 in.
8. 58740 ft. ; 140 ft. 2 in.

Ex. XXIII, p. 23.

1. 15863333 sq. inches.
2. 111955363 sq. inches.
3. 30528581 sq. inches.
4. 1376127 perches 11¼ yds.
5. 451 ac. 2 r. 35 p. 8¼ yds.
6. 3105 p. 19 yds. 4 ft. 64 in.
7. 390 r. 13 p. 8 yds. 1¾ ft.
8. 21782357 sq. in.

Ex. XXIV, p. 24.

1. 405 cub. ft.; 836352 cub. in.
2. 402 cub. yd. 23 cub. ft. 1076 cub. in.; 22463 cub. in.
3. 699112 cub. in.; 216 cub. yds. 13 cub. ft. 850 cub. in.
4. 13574 cub. ft. 917 cub. in.; 241935 cub. in.

Ex. XXV, p. 24.

1. 2536 pints; 21435 galls. 1 qt. 1 pt.
2. 456 gills; 1216 galls.
3. 22934 galls. 2 qts.; 47274 galls. 2 qts.
4. 194 galls.; 2560 galls.

Ex. XXVI, p. 24.

1. 13505 days; 41256 hours.
2. 9910 days 19 h. 46 m.; 76 weeks 6 d. 8 h. 21 m. 18 sec.
3. 652028 secs.; 4 months 0 w. 3 d. 17 h. 46 m. 40 sec.
4. 1394 years 0 m. 3 w. 3 d. 4 h.; 32846400 secs.

Ex. XXVII, p. 25.

A.

1. £448 7*s.*; 531 half gui. and 4*s.* 6*d.* over.
2. 27 gui. and 20*s.* 6*d.* over; 3192 half crs.
3. 119 half gui. and 5*s.* 6*d.* over; 1146 crs. and 3*s.* over.
4. 12382 crs. and 1*s.* 4*d.* over; 18750 fourpenny pieces.
5. 75 crs. and 2*s.* 6*d.* over; 1092 sixpences and 4*d.* over.
6. 98532 fourpenny pieces; 2860 seven shilling pieces.
7. 273 yds. 3 qrs.; 1000 yds.

B.

1. 140 lbs. 11 oz. 13 dwt. 8 qrs.; 18 lbs. 2 oz. 8 dwt.
2. 93 lbs. and 5640 grs. Troy over; 24 lbs. and 2100 grs. Troy over.

Ex. XXVIII, p. 26.

1. £12 5*s.* 4*d.*
2. £163 1*s.* 11*d.*
3. £137 1*s.* 8*d.*
4. £132 17*s.* 10*d.*
5. £115 10*s.* 4*d.*
6. £85 4*s.* 1¾*d.*
7. £139 19*s.* 6½*d.*
8. £189 12*s.* 7¾*d.*
9. £74 2*s.* 9*d.*
10. £155 13*s.* 10*d.*
11. £82 18*s.* 5*d.*
12. £81 14*s.* 5¾*d.*

13. 53 lbs. 2 oz. 13 drs.
14. 9 qrs. 9 lbs. 7 oz.
15. 61 cwt. 3 qrs. 3 lbs.
16. 81 qrs. 17 lbs. 3 oz.
17. 10 qrs. 12 lbs. 3 oz. 6 drs.
18. 122 cwt. 2 qrs. 27 lbs. 12 oz.
19. 41 tons 15 cwt. 3 qrs. 7 lbs. 10 oz.
20. 29 oz. 16 dwt. 12 grs.
21. 82 lbs. 6 oz. 11 dwt.
22. 35 oz. 16 dwt. 12 grs.
23. 159 lbs. 11 oz. 5 dwt.
24. 115 lbs. 4 oz. 0 dwt. 16 grs.
25. 107 lbs. 0 oz. 11 dwt. 18 grs.
26. 75 lbs. 4 oz. 17 dwt. 2 grs.
27. 22 drs. 1 scr. 16 grs.
28. 40 oz. 5 drs. 0 scr.
29. 33 drs. 2 scr. 13 grs.
30. 40 oz. 5 drs. 1 scr.
31. 61 yds. 0 ft. 3 in.
32. 23 fur. 34 p. 4 yds.
33. 20 miles 0 fur. 54 yds.
34. 24 lea. 0 m. 2 fur.
35. 19 fur. 19 p. 3½ yds.
36. 123 p. 2 yds. 0 ft.
37. 23 yds. 2 ft. 1 in.
38. 27 p. 3 yds. 34 in.
39. 29 p. ½yd. 2 ft. 0 in.
40. 118 m. 0 fur. 19 p. 1 yd.
41. 25 m. 1 fur. 17 yds. 20 in.
42. 153 yds. 0 qrs. 1 nail.
43. 984 yds. 1 qr. 0 nail.
44. 319 ells 4 qrs. 1 nail.
45. 176 ells 0 qrs. 3 nails.
46. 144 sq. yds. 2 sq. ft. 2 sq. in.
47. 59 r. 20 p. 17¾ sq. yds.
48. 104 a. 0 r. 35 p.
49. 29 p. ¾ sq. yd. 4 ft. 27 sq. in.
50. 102 a. 1 r. 4 p. 7¼ sq. yds.
51. 100 r. 30 p. 27¼ sq. yds. 390 sq. in.
52. 211 c. yds. 24 c. ft. 673 c. in.
53. 314 c. yds. 16 c. ft. 178 c. in.
54. 200 galls. 0 qts. 1 pt.
55. 109 bush. 2 pks. 0 galls.
56. 150 qrs. 1 bush. 3 pks. 1 gall.
57. 191 bush. 0 pks. 1 gall. 3 qts.
58. 61 dys. 20 hrs. 21 m. 51 secs.
59. 62 yrs. 180 dys. 0 hrs. 10 m.
60. 66 yrs. 5 wks. 3 dys. 8 hrs.

Ex. XXIX, p. 28.

1. £9 5*s.* 4*d.*
2. £17 4*s.* 1½*d.*
3. 13*s.* 1¼*d.*
4. £51 8*s.* 11*d.*
5. £4 9*s.* 10*d.*
6. £13 17*s.* 5¾*d.*
7. £44 17*s.* 1¾*d.*
8. £87 5*s.* 5¼*d.*
9. £105 14*s.* 3¾*d.*
10. £87 18*s.* 3¾*d.*
11. £26 16*s.* 0¾*d.*
12. £348 3*s.* 2½*d.*
13. 12 lbs. 3 oz. 13 drs.
14. 7 qrs. 15 lbs. 4 oz.
15. 24 cwt. 1 qr. 19 lbs.
16. 9 oz.
17. 4 lbs. 3 oz. 15 drs.
18. 8 tons 6 cwt. 2 qrs.
19. 4 cwt. 0 qrs. 24 lbs.
20. 9 cwt. 108 lbs. 5 oz.
21. 4 oz. 3 dwt. 10 grs.
22. 4 oz. 17 dwt. 23 grs.
23. 6 lbs. 7 oz. 17 dwt.
24. 8 oz. 1 dwt. 2 grs.
25. 11 oz. 4 dwt. 8 grs.
26. 26 oz. 8 dwt. 11 grs.
27. 8 oz. 9 dwt. 14 grs.
28. 6 oz. 6 dwt. 5 grs.
29. 2 drs. 0 scr. 19 grs.
30. 7 oz. 1 dr. 1 scr.
31. 18 lbs. 8 oz. 7 drs.
32. 17 grs.
33. 3 yds. 1 ft. 7 in.
34. 9 p. 12 yds. 2 ft.
35. 2 fur. 11 p. 2 yds.
36. 5 m. 7 fur. 215 yds.
37. 2 m. 1 fur. 27 p.
38. 38 p. 4 yds.
39. 8 lea. 2 m. 2 fur.
40. 17 p. 11 yds.

41. 9 p. 3 yds. 4 ft.
42. 16 yds. 0 ft. 3 in.
43. 1 yd. 3 qrs. 1 nail.
44. 3 ells 3 qrs. 2 nails.
45. 6 sq. yds. 2 sq. ft. 61 sq. in.
46. 9 p. 23 sq. yds. 7 sq. ft.
47. 1 a. 137 p. 18 sq. yds.
48. 3 a. 1 r. 31 p.
49. 23 a. 2 r. 27 p.
50. 1 r. 38 p. 23 sq. yds.
51. 4 r. 2$\frac{1}{4}$ sq. yds. 7 sq. ft.
52. 10 sq. yds. 3 sq. ft. 10 sq. in.
53. 5 c. yds. 16 c. ft. 643 c. in.
54. 65 c. yds. 4 c. ft. 628 c. in.
55. 31 c. yds. 21 c. ft. 1558 c. in.
56. 3 c. yds. 14 c. ft. 998 c. in.
57. 12 hrs. 56 m. 43 secs.
58. 7 dys. 18 hrs. 51 m.
59. 11 yrs. 185 dys. 21 hrs.
60. 3 yrs. 12 wks. 2 dys.

Ex. XXX, p. 30.

1. £10 16*s.* 6*d.*
2. £83 0*s.* 6*d.*
3. £119 0*s.* 9*d.*
4. £51 13*s.* 8*d.*
5. £176 6*s.* 4*d.*
6. £295 7*s.* 1*d.*
7. £499 3*s.* 4*d.*
8. £288 6*s.* 0$\frac{1}{2}$*d.*
9. £1282 1*s.* 0*d.*
10. £873 8*s.* 1$\frac{1}{2}$*d.*
11. £1003 6*s.* 1*d.*
12. £1131 15*s.* 11$\frac{1}{2}$*d.*
13. £845 17*s.* 4*d.*
14. £1103 7*s.* 6*d.*
15. £1042 17*s.* 6*d.*
16. £1691 15*s.* 10$\frac{1}{2}$*d.*
17. £2141 18*s.* 4*d.*
18. £1572 18*s.* 11$\frac{1}{2}$*d.*
19. £1271 15*s.* 8*d.*
20. £174 17*s.* 6$\frac{1}{2}$*d.*

Ex. XXXI, p. 30.

1. £220 8*s.* 8*d.*
2. £1364 0*s.* 10$\frac{1}{2}$*d.*
3. £397 11*s.* 6$\frac{3}{4}$*d.*
4. £1394 10*s.* 7$\frac{1}{2}$*d.*
5. £2225 14*s.* 2*d.*
6. £3357 3*s.* 1$\frac{1}{2}$*d.*
7. £1298 8*s.* 10*d.*
8. £2602 15*s.* 8*d.*
9. £1909 8*s.* 1$\frac{1}{4}$*d.*
10. £11596 12*s.* 3*d.*
11. £6368 17*s.*
12. £1918 15*s.*

Ex. XXXII, p. 31.

1. £43 8*s.* 1$\frac{1}{7}$*d.*
2. £28 14*s.* 0$\frac{3}{5}$*d.*
3. £90 11*s.* 5$\frac{5}{6}$*d.*
4. £15 19*s.* 10$\frac{5}{8}$*d.*
5. £63 3*s.* 6$\frac{2}{3}$*d.*
6. £124 2*s.* 8$\frac{1}{7}$*d.*
7. £144 10*s.* 9$\frac{3}{5}$*d.*
8. £97 14*s.* 1$\frac{1}{11}$*d.*

Ex. XXXIII, p. 32.

1. £425 17*s.* 6$\frac{1}{2}$*d.*
2. £544 18*s.* 9$\frac{3}{4}$*d.*
3. £1512 17*s.* 5$\frac{1}{2}$*d.*
4. £4251 8*s.* 2*d.*
5. £1993 4*s.* 2$\frac{1}{4}$*d.*
6. £4570 13*s.* 7$\frac{3}{4}$*d.*
7. £1568 18*s.* 4$\frac{1}{2}$*d.*
8. £5226 3*s.* 6*d.*
9. £39935 19*s.* 2$\frac{1}{4}$*d.*
10. £755 14*s.* 0$\frac{3}{4}$*d.*
11. 18 qrs. 7 lbs. 15 oz. 8 drs.
12. 51 tons 7 cwt. 52 lbs. 4 oz.
13. 87 tons 8 cwt. 7 lbs. 5 oz. 4 drs.
14. 315 tons 11 cwt. 1 qr. 0 lbs.
15. 417 cwt. 1 qr. 20 lbs. 8 oz. 2 drs.
16. 378 t. 14 c. 1 qr. 26 lb. 11 oz. 14 dr.
17. 136 lbs. 2 oz. 17 dwt. 12 grs.
18. 228 lbs. 0 oz. 19 dwt. 3 grs.
19. 231 fur. 3 yds. 0 ft. 4 in.
20. 615 fur. 18 yds. 2 ft. 6 in.
21. 350 a. 2 r. 8 p.
22. 1287 a. 3 r. 12 p.
23. 331 sq. yds. 2 sq. ft. 66 sq. in.
24. 424 c. yds. 2 c. ft. 1350 c. in.
25. 8491 gall. 2 qts. 0 pts.
26. 2489 yrs. 227 dys. 0 hrs. 24 min.

Ex. XXXIV, p. 34.

1. £7 7*s.* $1\frac{3}{4}$*d.* 2. £5 4*s.* 9*d.* 3. £18 3*s.* 11*d.*
4. £14 15*s.* $5\frac{1}{4}$*d.* 5. £13 5*s.* $0\frac{1}{4}$*d.* 6. £4 7*s.* $5\frac{3}{4}$*d.*
7. £4 16*s.* 9*d.* 8. £10 17*s.* 8*d.* 9. £1 7*s.* $7\frac{1}{4}$*d.*
10. £4 7*s.* 2*d.* 11. £8 15*s.* $10\frac{1}{2}$*d.* 12. £23 1*s.* $7\frac{3}{4}$*d.*

Ex. XXXV, p. 35.

1. 15*s.* $7\frac{5}{23}$*d.* 2. £3 7*s.* $6\frac{8}{146}$*d.* 3. £1 3*s.* $2\frac{65}{136}$*d.*
4. £6 1*s.* $10\frac{11}{152}$*d.* 5. £9 10*s.* $3\frac{173}{203}$*d.* 6. £6 1*s.* $4\frac{26}{413}$*d.*
7. 2 qrs. 6 lb. $0\frac{37}{147}$ oz. 8. 1 qr. 1 lb. 9 oz. $10\frac{2773}{2968}$ drs.
9. 4 cwt. 0 qrs, 9 lbs. $15\frac{76}{370}$ oz. 10. 2 lbs. 15 oz. $5\frac{6641}{7370}$ drs.
11. 4 m. 2 fur. $19\frac{108}{119}$ p. 12. 5 fur. $4\frac{20}{137}$ p.
13. 4 dys. 11 hrs. $0\frac{28}{246}$ min. 14. 2 dys. 6 hrs. 31 min. $45\frac{243}{317}$ secs.
15. 1 a. 2 r. $9\frac{1}{193}$ p. 16. 3 p. 2 yds. $0\frac{1049}{1754}$ ft.
17. 4*s.* $4\frac{8389}{13986}$*d.* 18. 3*s.* $5\frac{1873}{4113}$*d.*

Ex. XXXVI, p. 35.

1. £35 7*s.* 4*d.* 2. £6 1*s.* 3*d.* 3. £9 17*s.* $5\frac{25}{100}$*d.*
4. £2 18*s.* 11*d.* 5. £7 11*s.* $1\frac{750}{1000}$*d.* 6. 5*s.* 10*d.*
7. 12*s.* 11*d.* 8. £21 5*s.*

Ex. XXXVII, p. 36.

1. 13 times. 2. 12 times. 3. 9 times.
4. 27 ,, 5. 8 ,, 6. 132000 times.
7. 16 ,, 8. 107 ,, 9. 243 times.
10. 7 ,,

MISCELLANEOUS EXAMPLES, pp. 36, 37.

1. 98240 halfpence. 2. £22652 9*s.* $8\frac{1}{4}$*d.* 3. 74000 oz.
4. 443 cwt. 1 qr. 14 lb. 7 oz. 5. 9707 tons 17 cwt. 1 qr. 16 lbs.
6. 11554 qrs. 7. 135510 in. 8. 214181 in.
9. 82348 m. 7 fur. 39 p. 2 yds. 0 ft. 4 in.
10. 547 m. 7 fur. 14 p. 4 yds. 0 ft. 8 in.
11. 17377 sixpences. 12. 204116 gui. 5*s.* 6*d.* 13. 26756729 sq. in.
14. 138 p. 16 sq. yds. 2 ft. 80 in. 15. 12309303 sq. in.
16. 4 a. 3 r. 37 p. 7 yds. 0 ft. 61 in. 17. 3033 c. yds. 9 c. ft. 676 c. in.
18. 794915 c. in. 19. £4951 16*s.* 20. 297727 half crs.
21. 35 wks. 4 dys. 8 hrs. 23 m. 6 secs. 22. 832092 secs.
23. 1744 pts. 24. 330 galls. 25. 192901 qrs. 1 bush. 3 pks.
26. 2728 grs. 27. 1783 lbs. 28. 2631 cwt. 1 qr. 4 lbs. 4 oz.
29. 595421 yds. 1 ft. 8 in. 30. £396 18*s.* 31. £170 18*s.* 4*d.*

32. £115 1*s.* $0\frac{1}{2}$*d.*   33. 11 cwt. 2 qrs. 12 lbs. 0 oz. 2 drs.
34. 165 cwt. 2 qrs. 1 lb. 1 oz.   35. £1634 3*s.* 3*d.*   36. £2 5*s.* $2\frac{1}{4}$*d.*
37. £2 3*s.* $0\frac{8}{9}$*d.*   38. 14*s.* $9\frac{73}{126}$*d.*   39. £4 16*s.* $8\frac{124}{273}$*d.*
40. 16*s.* $6\frac{5}{6}$*d.*   41. £1200 5*s.* $7\frac{1}{2}$*d.*   42. £25 16*s.* $3\frac{6}{7}$*d.*
43. £21 4*s.* $6\frac{2}{3}$*d.*   44. 27 cwt. 0 qrs. $8\frac{2}{3}$ lbs.   45. 89 qrs. 11 lbs. $2\frac{1}{2}$ oz.
46. 5*s.* $0\frac{71}{84}$*d.*   47. 2*s.* $2\frac{236}{1584}$*d.*   48. 4*s.* $10\frac{67}{160}$*d.*
49. £1 15*s.* $4\frac{1345}{1452}$*d.*   50. £5 11*s.* $7\frac{31}{42}$*d.*

Ex. XXXVIII, p. 39.

1. 20 sq. yds.   2. 15 sq. yds. 2 sq. ft. 36 sq. in.
3. 156 sq. yds.   4. 21 sq. yds. 7 sq. ft.

Ex. XXXIX, p. 39.

1. 37 ft. $2\frac{1}{33}$ in.   2. 70 yds. 2 ft. 8 in.
3. 4 ft. $11\frac{27}{37}$ in.   4. 7 yds. 1 ft. $4\frac{16}{27}$ in.

Ex. XL, p. 39.

1. 2 ft. 9 in.   2. 31 ft. 7 in.   3. 3 ft. 1 in.   4. 22 ft. 4 in.

Ex. XLI, pp. 39, 40, 41.

1. 402 sq. yds. 8 sq. ft.   2. 16 ft. 5 in.   3. 30 ft. 6 in.
4. 10 ft. 4 in.   5. $573\frac{1}{6}$ sq. ft.   6. 704 sq. ft.
7. 155 sq. ft.   8. $134\frac{2}{3}$ yds.   9. 16 ft. 6 in.
10. $35\frac{4}{39}$ yds.   11. $67\frac{5}{6}$ yds.   12. $110\frac{2}{3}$ yds.
13. 93 yds.   14. $24\frac{2}{3}$ yds.

Ex. XLII, p. 43.

1. 20 c. yds. 12 c. ft. 594 c. in.   2. 5 c. yds. 2 c. ft. 1296 c. in.
3. 4 c. yds. 3 c. ft. 582 c. in.   4. 7 ft. 5 in.   5. 12 ft.
6. 3 ft. 4 in.   7. $1313\frac{7}{36}$ cub. ft.   8. 378 cub. ft
9. 116 ft.   10. 432 cub. ft.   11. $2\frac{1}{2}$ ft.   12. $41\frac{1}{3}$ ft.
13. $42\frac{13}{27}$ sq. yds.; £7 8*s.* $8\frac{2}{3}$*d.*   14. £18 6*s.* 8*d.*

Ex. XLIII, p. 44.

1. 112.   2. 2.   3. 1.   4. 25.
5. 303.   6. 381.   7. 258.   8. 18.
9. 35.   10. 25.   11. 1728.   12. 515.

Ex. XLIV, p. 44.

1. 4.   2. 112.   3. 19.
4. 5.   5. 173.   6. 1.

Ex. XLV, p. 45.

1. 100. 2. 84. 3. 24. 4. 462.
5. 1680. 6. 840. 7. 72. 8. 360.
9. 420. 10. 168. 11. 5040. 12. 2520.
13. 1980. 14. 10080.

Ex. XLVI, p. 47.

1. $\frac{63}{7}$ $\frac{189}{7}$; $\frac{99}{11}$ $\frac{297}{11}$. 2. $\frac{481}{13}$ $\frac{1651}{13}$; $\frac{629}{17}$ $\frac{2159}{17}$. 3. $\frac{41}{8}$ $\frac{106}{13}$ $\frac{196}{15}$ $\frac{211}{10}$.
4. $\frac{64}{3}$ $\frac{449}{12}$ $\frac{659}{9}$ $\frac{1233}{11}$. 5. $\frac{562}{5}$ $\frac{156}{4}$. 6. $\frac{1270}{10}$.

Ex. XLVII, p. 48.

1. $4\frac{2}{3}$. 2. $7\frac{3}{5}$. 3. $17\frac{7}{12}$. 4. $155\frac{8}{11}$.
5. $353\frac{7}{9}$. 6. $116\frac{1}{7}$. 7. 127. 8. $57\frac{29}{37}$.
9. $796\frac{4}{19}$. 10. $180\frac{87}{117}$. 11. $190\frac{128}{711}$. 12. $151\frac{419}{999}$.

Ex. XLVIII, p. 49.

1. $\frac{5}{24}$. 2. $\frac{1}{112}$. 3. $2\frac{2}{19}$. 4. $\frac{2}{9}$.
5. $\frac{27}{40}$. 6. $1\frac{1}{27}$. 7. $\frac{5}{11}$. 8. $3\frac{1}{27}$.
9. $35\frac{155}{224}$. 10. $\frac{25}{32}$. 11. $39\frac{31}{136}$. 12. $37\frac{71}{432}$.
13. $262\frac{1}{2}$. 14. $6\frac{62}{63}$. 15. $\frac{1}{4}$. 16. $80\frac{53}{140}$.

Ex. XLIX, p. 49.

1. $\frac{9}{20}$. 2. $\frac{5}{24}$. 3. $1\frac{7}{11}$. 4. $\frac{4}{13}$. 5. $\frac{40}{143}$. 6. $\frac{87}{131}$.
7. $\frac{9}{22}$. 8. $\frac{113}{144}$. 9. $\frac{13}{14}$. 10. $\frac{158}{367}$. 11. $\frac{32}{97}$. 12. $\frac{3}{4}$.
13. $\frac{11}{16}$. 14. $\frac{9}{32}$. 15. $\frac{60}{77}$. 16. $\frac{77}{81}$.

Ex. L, p. 50.

1. $\frac{5}{9}$. 2. $\frac{9}{11}$. 3. $\frac{2}{9}$. 4. $\frac{3}{11}$, 5. $\frac{7}{9}$. 6. $\frac{112}{113}$.
7. $\frac{9}{11}$. 8. $\frac{9}{11}$. 9. $\frac{5}{11}$. 10. $\frac{4}{11}$. 11. $\frac{21}{23}$. 12. $\frac{3}{7}$.

Ex. LI, p. 51.

1. $\frac{10}{30}$, $\frac{12}{30}$, $\frac{5}{30}$, $\frac{6}{30}$. 2. $\frac{35}{210}$, $\frac{150}{210}$, $\frac{126}{210}$, $\frac{140}{210}$.
3. $\frac{40}{60}$, $\frac{45}{60}$, $\frac{48}{60}$, $\frac{50}{60}$. 4. $\frac{54}{144}$, $\frac{64}{144}$, $\frac{9}{144}$, $\frac{136}{144}$.
5. $\frac{24}{32}$, $\frac{12}{32}$, $\frac{26}{32}$, $\frac{29}{32}$. 6. $\frac{96}{120}$, $\frac{100}{120}$, $\frac{105}{120}$, $\frac{108}{120}$.
7. $\frac{63}{144}$, $\frac{40}{144}$, $\frac{78}{144}$, $\frac{51}{144}$. 8. $\frac{81}{405}$, $\frac{90}{405}$, $\frac{195}{405}$, $\frac{25}{405}$.
9. $\frac{120}{420}$, $\frac{42}{420}$, $\frac{245}{420}$, $\frac{204}{420}$. 10. $\frac{440}{1080}$, $\frac{585}{1080}$, $\frac{180}{1080}$, $\frac{144}{1080}$, $\frac{120}{1080}$.
11. $\frac{36}{450}$, $\frac{270}{450}$, $\frac{18}{450}$, $\frac{195}{450}$, $\frac{130}{450}$. 12. $\frac{1620}{3780}$, $\frac{3465}{3780}$, $\frac{1008}{3780}$, $\frac{1120}{3780}$, $\frac{864}{3780}$.

Ex. LII, p. 51.

1. 2. 2. $1\frac{13}{21}$. 3. $2\frac{11}{72}$. 4. $1\frac{59}{80}$. 5. $2\frac{35}{36}$.
6. $1\frac{229}{1008}$. 7. $1\frac{89}{90}$. 8. $1\frac{131}{210}$. 9. $7\frac{1}{12}$. 10. $8\frac{53}{56}$.
11. $12\frac{83}{315}$. 12. $11\frac{107}{144}$. 13. $1\frac{253}{315}$. 14. $5\frac{29}{63}$. 15. $32\frac{59}{140}$.
16. $20\frac{161}{344}$. 17. $75\frac{1}{3}$. 18. $14\frac{2}{3}$. 19. $18\frac{1}{7}$. 20. $27\frac{115}{252}$.

Ex. LIII, p. 52.

1. $\frac{5}{6}$, $\frac{4}{7}$, $\frac{5}{11}$, $\frac{1}{12}$. 2. $\frac{32}{33}$, $1\frac{25}{36}$, $\frac{6}{7}$, $9\frac{5}{6}$. 3. $\frac{29}{100}$, $7\frac{1}{17}$, $17\frac{3}{4}$, $2\frac{17}{28}$.
4. $8\frac{25}{26}$, $4\frac{9}{1000}$, $4\frac{101}{126}$, $1\frac{481}{483}$. 5. $11\frac{34}{105}$, $849\frac{447}{512}$. 6. $3\frac{2}{7}$, $3\frac{37}{108}$.

Ex. LIV, p. 53.

1. $1\frac{23}{42}$. 2. $25\frac{1}{336}$. 3. $17\frac{67}{80}$. 4. $9\frac{25}{144}$.
5. $59\frac{157}{384}$. 6. $1\frac{211}{432}$.

Ex. LV, p. 53.

1. $\frac{33}{70}$; $119\frac{3}{7}$; $25\frac{7}{8}$. 2. $235\frac{1}{8}$; $25\frac{95}{196}$; $7\frac{1}{2}$. 3. $32\frac{13}{16}$; $10\frac{19}{99}$; $34\frac{119}{120}$.
4. $1\frac{47}{756}$; $16\frac{19}{21}$; $38\frac{4}{5}$. 5. $5\frac{61}{135}$; $103\frac{13}{45}$.

Ex. LVI, p. 55.

1. $\frac{9}{10}$; $\frac{8}{9}$; $\frac{27}{32}$; $11\frac{11}{24}$. 2. $2\frac{19}{22}$; $\frac{22}{63}$; $1\frac{9}{11}$; $\frac{11}{20}$. 3. $31\frac{1}{2}$; $2\frac{206}{2025}$; $83\frac{29}{49}$.
4. $2\frac{11}{32}$; $\frac{1}{4}$; 4; $1\frac{33}{82}$; $\frac{28}{33}$. 5. $1\frac{6}{17}$; $4\frac{29}{40}$: $3\frac{21}{92}$; $\frac{28}{297}$. 6. $\frac{1506}{2177}$; $15\frac{1119}{1540}$.
7. $1\frac{259}{500}$; $1\frac{43}{93}$. 8. $\frac{35}{121}$; $9\frac{243}{1085}$.

Ex. LVII, p. 56

1. $\frac{11}{96}$; $\frac{89}{108}$. 2. $\frac{259}{1142}$; $\frac{47}{168}$. 3. $\frac{71}{476}$; $\frac{9}{28}$. 4. $\frac{17}{240}$; $\frac{1119}{7168}$.
5. $\frac{527}{1108}$; $\frac{59}{137}$. 6. $\frac{109}{400}$; $\frac{20}{21}$. 7. $1\frac{51}{89}$; $1\frac{5}{8}$. 8. $\frac{445}{672}$; $5\frac{28}{33}$.
9. $\frac{147}{3200}$; $\frac{478}{585}$. 10. $\frac{1}{9}$; $\frac{43}{100}$. 11. $\frac{20}{21}$; $1\frac{3}{4}$. 12. $\frac{11}{80}$; $19\frac{3}{5}$.
13. $\frac{1}{360}$; $\frac{11}{90}$. 14. $1\frac{4}{45}$; $\frac{25}{168}$.

Ex. LVIII, p. 56.

1. £19 8*s.* $5\frac{1}{4}$*d.* 2. £34 7*s.* $2\frac{9}{14}$*d.* 3. £25 0*s.* $0\frac{7}{24}$*d.*
4. £37 13*s.* $3\frac{11}{20}$*d.* 5. £32 14*s.* $9\frac{3}{4}$*d.* 6. £21 12*s.* $9\frac{1}{8}$*d.*
7. £119 18*s.* $8\frac{2}{3}$*d.* 8. £80 19*s.* $4\frac{5}{8}$*d.* 9. £95 7*s.* $2\frac{5}{8}$*d.*

Ex. LIX, p. 57.

1. £6 11*s.* $10\frac{5}{6}$*d.* 2. £1 10*s.* $8\frac{7}{8}$*d.* 3. £2 17*s.* $11\frac{1}{24}$*d.*
4. £11 2*s.* $7\frac{5}{24}$*d.* 5. £1 18*s.* $10\frac{13}{48}$*d.* 6. £3 18*s.* $11\frac{1}{12}$*d.*
7. £2 1*s.* $7\frac{5}{8}$*d.* 8. 6*s.* $9\frac{7}{8}$*d.*

Ex. LX, p. 58.

1. 12*s.*; 35*s.*; 4*s.* 5$\frac{1}{3}$*d.*; £2 17*s.* 9$\frac{1}{3}$*d.*
2. £1 11*s.* 8$\frac{4}{5}$*d.*; £6 10*s.* 0$\frac{1}{2}$*d.*; £12 4*s.* 8$\frac{1}{5}$*d.*
3. £3 4*s.* 9$\frac{1}{7}$*d.*; £2 17*s.* 9$\frac{2}{3}$*d.*; £6 4*s.* 10$\frac{3}{49}$*d.*
4. £56 13*s.* 2$\frac{7}{16}$*d.*; £39 17*s.* 0$\frac{27}{32}$*d.*; £20 7*s.* 1$\frac{3}{16}$*d.*
5. £118 5*s.* 4$\frac{3}{8}$*d.*; £253 9*s.* 2*d.*; £33 4*s.* 9$\frac{13}{16}$*d.*
6. 8 cwt. 2 qrs. 8 lbs.; 5 oz. 2 dwts. 20$\frac{4}{7}$ grs.; 4 cwt. 1 qr. 9 lbs. 5 oz. 5$\frac{1}{3}$ drs.
7. 6 dys. 4$\frac{1}{2}$ hrs.; 11 wks. 1 dy. 5$\frac{1}{3}$ hrs.; 3*s.* 7$\frac{21}{32}$*d.*
8. 6*s.* 10$\frac{1}{2}$*d.*; 11 cwt. 3 qrs. 22$\frac{13}{24}$ lbs.; £49 6*s.* 10$\frac{2}{7}$*d.*
9. 19*s.* 3*d.*; 16*s.* 9*d.*; £1 13*s.* 11$\frac{1}{13}$*d.*
10. £2 15*s.* 11$\frac{5}{8}$*d.*; 12*s.* 11$\frac{1}{4}$*d.*; £2 0*s.* 8$\frac{1}{2}$*d.*
11. £1 2*s.* 7$\frac{1}{2}$*d.*; 12*s.* 8*d.*
12. £1 9*s.* 11*d.*; £7 18*s.* 1$\frac{1}{7}$*d.*

MISCELLANEOUS QUESTIONS, XLIII—LX, pp. 58, 59, 60.

1. $\frac{45,\ 40,\ 48}{150}$ 2. $\frac{133}{150}$. 3. $\frac{16}{625}$. 4. 840.
5. 2. 6. 8$\frac{11}{24}$. 7. $\frac{119}{200}$.
8. 11 cwt. 0 qrs. 6lbs. 9. £17 18*s.* 8$\frac{2}{7}$*d.* 10. 2*s.* 7$\frac{193}{351}$*d.*
11. $\frac{4}{5}$ greatest $\frac{9}{125}$ least. 12. £5 8*s.* 13. latter by $\frac{7}{16}$.
14. £2 10*s.* 5*d.* 15. £17 3*s.* 11$\frac{1}{20}$*d.* 16. 6*s.* 3$\frac{259}{295}$*d.*
17. $\frac{89}{426}$. 18. £24 19*s.* 5$\frac{17}{60}$*d.* 19. £81 14*s.* 3$\frac{5}{6}$*d.* 20. $\frac{101}{485}$.
21. 7$\frac{55}{56}$. 22. 19$\frac{9}{88}$. 23. 12$\frac{822}{858}$. 24. 7$\frac{374}{1141}$. 25. 173$\frac{4}{9}$.
26. 2082$\frac{2}{9}$. 27. 133$\frac{33}{40}$. 28. 1$\frac{16}{21}$. 29. 13$\frac{1}{12}$. 30. 1$\frac{3}{8}$.
31. 26$\frac{17}{108}$ sq. yds. 32. 16 ft. 33. 959 sq. ft. 63 sq. in.
34. 16 ft. 35. 346$\frac{5}{32}$ cub. ft. 36. 24 ft. 37. 23$\frac{19}{42}$.
38. $\frac{21489}{20120}$. 39. $\frac{3303}{16625}$. 40. 1$\frac{449}{976}$. 41. $\frac{5}{12}$.
42. 6500 times. 43. £285 15*s.* 9$\frac{6}{7}$*d.* 44. £11 2*s.* 45. $\frac{3315}{4459}$.
46. 257 galls. 0 qts. 0$\frac{2}{13}$ pts. 47. 59 yds. 48. $\frac{495}{2744}$.
49. 120 yds.; 25 s. 50. 3 tons 0 cwt. 2 qrs. 13 lbs.

Ex. LXI, p. 62.

1. 1472·15; 14·7215. 2. 15250·; 1·525.
3. 1205120·5; 1·2051205. 4. ·01325. 5. 5·; ·05; 50·; ·005.
6. 10 × 10 × 10. 7. Twice. 8. 10000.

Ex. LXII, p. 62.

1. ·01; ·7; ·113; ·000011. 2. ·3; 10·3; ·000017.
3. 12·18; ·001218. 4. ·0009; ·009; 1·9.
5. 101·9; 10·19; 1·019.

Ex. LXIII, p. 63.

1. $\frac{107}{1000}$; $\frac{17}{100}$; $\frac{17}{10000}$; $\frac{1219}{100000}$. 2. 3$\frac{9}{10}$; 4$\frac{7}{40}$; 5$\frac{27}{200}$; 5$\frac{117}{1000}$.
3. $\frac{1}{10000}$; $\frac{17}{1000}$; $\frac{1}{200000}$; $\frac{1}{1000}$. 4. 15$\frac{1}{4}$; 18$\frac{1}{5}$; 134$\frac{1}{4}$; 53$\frac{31}{200}$.
5. 8$\frac{1}{8}$; 10$\frac{1}{200}$; 15$\frac{1}{20}$; 7$\frac{3}{2000}$. 6. 10$\frac{1}{10}$; 100$\frac{1}{1000}$; 7$\frac{21}{200}$; 6$\frac{11}{2000}$.

Ex. LXIV, p. 63.

1. 52·36925. 2. 41707·3607. 3. 1123·2232.
4. 32·17514. 5. 35·74852. 6. 16803·87625.

Ex. LXV, p. 64.

1. 3·225. 2. 1·498675. 3. 170·97989. 4. ·252735.
5. 1·0788. 6. 2158·34832. 7. ·06012. 8. ·006.
9. ·0000375. 10. 12·01595. 11. 3·2593. 12. 1·55776.

Ex. LXVI, p. 64.

1. 236·375. 2. 10·5625. 3. ·000005. 4. ·00000025.
5. 522·875. 6. ·00875. 7. ·65. 8. ·0000024.
9. 1023·75. 10. ·0000003375. 11. 138·8475. 12. ·301875.

Ex. LXVII, p. 65.

1. 26·251. 2. 445·715. 3. 15·001. 4. 39·678.
5. 25·1872. 6. 11·1112. 7. 37·5015. 8. ·04585.
9. ·010868 +.

Ex. LXVIII, p. 66.

1. ·25. 2. ·0125. 3. 6250·. 4. 625·.
5. 980. 6. 490000·. 7. ·085. 8. 8·5.
9. 3·4. 10. 12·696652 +. 11. 1000·. 12. 100000·.

Ex. LXIX, p. 66.

1. ·25; ·5; ·75; ·15. 2. ·824; 11·923076; ·12; 3·0625.
3. 4·916666 +; ·957627 +; ·008547 +; 12·6.
4. ·004651 +; ·000999 +; ·012935 +; ·005327 +.
5 13·142857 +; 12·375; 10·05; 5·8.
6. 3·035714 +; 4·011428 +; 5·9375; 8·091666 +.
7. 33·515625. 8. ·15.

Ex. LXX, p. 67.

1. $\frac{37}{99}$; $\frac{5}{33}$; $\frac{119}{999}$; $3\frac{34}{99}$; $5\frac{127}{999}$. 2. $4\frac{8}{99}$; $5\frac{8}{9}$; $13\frac{1}{33}$; $7\frac{8}{999}$; $3\frac{4}{99}$.
3. $14\frac{14}{99}$; $7\frac{23}{99}$; $\frac{13}{111}$; $4\frac{26}{99}$; $19\frac{19}{99}$.

Ex. LXXI, p. 68.

1. $\frac{307}{900}$; $3\frac{206}{495}$; $1\frac{7}{45}$; $4\frac{8}{225}$. 2. $2\frac{331}{450}$; $5\frac{263}{990}$; $15\frac{117}{900}$; $3\frac{23}{90}$.
3. $1\frac{31}{2475}$; $7\frac{35}{111}$; $8\frac{437}{3330}$; $17\frac{7}{60}$. 4. $3\frac{983}{4500}$; $3\frac{1091}{4995}$; $3\frac{728}{3333}$; $3\frac{721}{3300}$.
5. $\frac{127}{3000}$; $\frac{419}{9900}$; $\frac{141}{3330}$; $4\frac{19}{450}$. 6. $4\frac{26}{225}$; $7\frac{71}{330}$; $6\frac{5}{33}$; $15\frac{1}{3}$.

Ex. LXXII, p. 68.

1. 28·173218. 2. 190·492929.
3. ·327777; ·058361; 2·874274; ·76.

Ex. LXXIII, p. 69.

1. $12{\cdot}0\dot{8}5\dot{1}$; $1{\cdot}11\dot{0}\dot{1}$; 4·713804 +. 2. $1{\cdot}217\dot{2}$; $12{\cdot}\dot{9}\dot{5}$; $1032{\cdot}\dot{6}$.
3. $3{\cdot}\dot{6}$; ·00625; ·759903 +. 4. 2·012276 +; 2·846153 +; 1·003569 +.

Ex. LXXIV, p. 70.

1. 7*s.*; 4*s.* 6*d.* 2. £8 4*s.* 6*d.*; 6*s.* $1\frac{1}{2}$*d.*
3. 17*s.* $3\frac{12}{23}$*d.*; 1*s.* $5\frac{1}{100}$*d.* 4. £2 3*s.* $5\frac{7}{10}$*d.*; £28 9*s.* 3*d.*
5. 18 c. 0 qrs. 14 lbs.; 3 t. 3 c. 3 qrs.
6. £13 6*s.* $0\frac{3}{4}$*d.*; £190 1*s.* $4\frac{1}{2}$*d.* 7. £1 2*s.* $0\frac{3}{5}$*d.*; £9 13*s.* 4*d.*
8. £6 17*s.* $9\frac{813}{1000}$*d.*; $2\frac{191}{200}$*d.* 9. $1642\frac{1}{2}$ yds.; 7 c. 0 qrs. 16 lbs.
10. 1 sq. ft. $86\frac{1}{23}$ sq. in.; 66 m. 0 f. $37\frac{1}{2}$ p.
11. £2 18*s.* $6\frac{39}{40}$*d.*; £16 5*s.* 10*d.* 12. £19 13*s.* $2\frac{7}{10}$*d.*; 18*s.* $2\frac{2}{3}$*d.*
13. 2*s.* $7\frac{5}{8}$*d.*; 2*s.* $7\frac{16}{25}$*d.* 14. £1 16*s.* $6\frac{3}{10}$*d.*
15. £1 17*s.* $7\frac{4}{5}$*d.*

Ex. LXXV, p. 71.

1. ·225; 1·175. 2. $1{\cdot}\dot{3}$; $\cdot72708\dot{3}$. 3. 1·175; ·22619+.
4. ·807692+; ·640668+. 5. ·835317+; ·094531+.
6. $\cdot3\dot{8}$; ·272159+. 7. ·048619+; ·346018+.
8. ·508928+; ·042465+. 9. $\cdot68541\dot{6}$; ·200148+.
10. $\cdot\dot{4}7619\dot{0}$; ·00558+. 11. 3·7; 1·460318+.
12. ·00625; 27·380952.

Miscellaneous Examples (LXI—LXXV), pp. 71, 72.

1. 1·7; 11·7; ·0003; 13·001. 2. ·4; ·16; ·02; ·007812+.
3. 226·0125. 4. 57·288. 5. 122276·84583.
6. 1953·125. 7. ·080645+. 8. 4·29518+.
9. $3\frac{1}{8}$; $3\frac{113}{900}$. 10. 6*s.* $3\frac{3}{50}$*d.* 11. ·964285+; $1{\cdot}\dot{0}3\dot{7}$.
12. 1*s.* $4\frac{3}{4}$*d.* 13. $1{\cdot}1\dot{6}$. 14. 9*s.* 2*d.*
15. ·216269+. 16. 4 h. 6 m. $57\frac{3}{5}$ s. 17. $2\frac{5}{16}$*d.*; £11 14*s.*
18. £7 0*s.* $7\frac{1}{2}$*d.* 19. £19 3*s.* 4*d.* 20. £10 19*s.* $4\frac{1}{2}$*d.*
21. £9 1*s.* 6*d.* 22. $3{\cdot}70\dot{3}7\dot{0}$.
23. £2 10*s.* $10\frac{47}{100}$*d.*; £12 6*s.* $10\frac{1}{5}$*d.*; $\cdot001\dot{6}$.
24. ·12; ·241379+; $4\frac{17}{100}$; $4\frac{8}{45}$. 25. 8960; £504.
26. $215\frac{7}{32}$ sq. ft. 27. £18 19*s.* $8\frac{29}{80}$*d.*; £1838 6*s.* 8*d.*
28. $1\frac{149}{200}$; 1·745. 29. 7·28 lbs.; £4 3*s.* $4\frac{3}{16}$*d.*
30. $\frac{13}{20}$; $\frac{13}{200}$; ·2953125. 31. $7\frac{14}{25}$*d.* 32. ·424528+. 33. 300.

34. 4 cwt. 0 qrs. 2 lbs.; 8344·7. 35. ·930952+.
36. ·003958+. 37. 84. 38. 2·131578+. 39. £2 12*s*. 6*d*.
40. ·733508+. 41. $\frac{139}{999}$; $\frac{113}{333}$; $18\frac{135}{1111}$. 42. $3\frac{4}{225}$; $15\frac{37}{3000}$.
43. $1\cdot\dot{2}$. 44. 6·3675. 45. 15·555549+.
46. £3 14*s*. $4\frac{1}{20}$*d*. 47. 5·6448 oz. 48. 30140 yds.
49. £3·28125. 50. $\cdot\dot{9}5238\dot{0}$.

Ex. LXXVI, p. 74.

1. £759. 2. £820. 3. £337 17*s*. 6*d*.
4. £904 3*s*. 4*d*. 5. £3564 15*s*. 6. £1341 12*s*.
7. £385. 8. £983 3*s*. 4*d*. 9. £574 15*s*.
10. £928 16*s*. 11. £1770 16*s*. 8*d*. 12. £1843 4*s*.

Ex. LXXVII, p. 75.

1. £1265 12*s*. 6*d*. 2. £1184 11*s*. 3. £259 3*s*.
4. £1421 8*s*. 5. £2452 6*s*. 6. £410 12*s*. 6*d*.
7. £3462 2*s*. 8. £1707 13*s*. 9. £327 13*s*. 6*d*.
10. £3753 18*s*. 11. 3108 12*s*. 12. £344 13*s*. 4*d*.

Ex. LXXVIII, p. 75.

1. £52 13*s*. $1\frac{1}{2}$*d*. 2. £238 9*s*. 8*d*. 3. £102 3*s*. 6*d*.
4. £170 1*s*. $0\frac{1}{2}$*d*. 5. £105 8*s*. $8\frac{1}{2}$*d*. 6. £46 9*s*. 3*d*.
7. £174 13*s*. 9*d*. 8. £686 9*s*. 2*d*. 9. £953 2*s*. 6*d*.
10. £1424 2*s*. $4\frac{1}{2}$*d*. 11. £343 18*s*. 7*d*. 12. £160 2*s*. 6*d*.

Ex. LXXIX, p. 76.

1. £446 7*s*. 2. £1208 14*s*. 3. £1561 11*s*. 4*d*.
4. £4083 4*s*. 5. £431 15*s*. 6*d*. 6. £870 2*s*.
7. £7288 14*s*. $8\frac{1}{2}$*d*. 8. £286 9*s*. 9*d*. 9. £811 7*s*. $3\frac{1}{2}$*d*.
10. £415 9*s*. 11. £1800 18*s*. $1\frac{1}{2}$*d*. 12. £1348 19*s*. 2*d*.

Ex. LXXX, p. 76.

1. £20 6*s*. $5\frac{4}{7}$*d*. 2. £17 0*s*. $11\frac{29}{56}$*d*. 3. £16 5*s*. $6\frac{27}{28}$*d*.
4. £21 6*s*. $6\frac{59}{64}$*d*. 5. £2 16*s*. $1\frac{123}{224}$*d*. 6. £3 10*s*. $8\frac{39}{112}$*d*.
7. £23 0*s*. $6\frac{1}{16}$*d*. 8. £10 17*s*. $4\frac{401}{448}$*d*. 9 £8 16*s*. $6\frac{3}{16}$*d*.
10. £13 15*s*. $1\frac{2}{3}$*d*. 11. £155 6*s*. $4\frac{1}{20}$*d*. 12. £7 9*s*. $2\frac{11}{14}$*d*.

MISCELLANEOUS QUESTIONS (LXXVI—LXXX), pp. 77, 78.

1. £390. 2. £756 19*s*. $10\frac{1}{2}$*d*. 3. £104 3*s*. $6\frac{1}{2}$*d*.
4. £4375. 5. £351. 6. £87 17*s*. $9\frac{3}{4}$*d*.

7. £284 19*s.* $9\frac{1}{2}$*d.*
8. £2270 8*s.* $2\frac{1}{2}$*d.*
9. 8 lbs. 3 oz. 12 dwt.
10. £28 5*s.* 6*d.*
11. £196 14*s.* 6*d.*
12. £10 14*s.* $11\frac{79}{112}$*d.*
13. $62\frac{2}{3}$ sq. yds., £4 17*s.* $2\frac{2}{3}$*d.*
14. £284 14*s.* 6*d.*
15. 437 cub. ft., 355 cwt. 0 qrs. 7 lbs.
16. £11 4*s.* $4\frac{3}{8}$*d.*
17. £16 10*s.* $1\frac{1}{2}$*d.*
18. £5802 12*s.* $2\frac{1}{4}$*d.*
19. £19 11*s.* $8\frac{47}{112}$*d.*
20. £2822 13*s.* 3*d.*

Ex. LXXXI, p. 79.

1. 108.
2. 36.
3. 9.
4. 27.
5. 96.
6. 144.
7. 45.
8. 15.

Ex. LXXXII, p. 79.

1. $\frac{1}{2}$.
2. $\frac{7}{11}$.
3. $\frac{1}{2}$.
4. $\frac{7}{64}$.
5. $\frac{423}{781}$.
6. $\frac{11}{30}$.
7. $\frac{54}{89}$.
8. $\frac{15}{16}$.

Ex. LXXXIII, p. 80.

1. 252 bushels.
2. 200 galls.
3. 240 ac.
4. 300 lbs.
5. £24.
6. 60 yds.

Ex. LXXXIV, p. 81.

1. 560 men.
2. 9 days.
3. $3\frac{3}{4}$ wks.
4. 11 horses.
5. $44\frac{2}{14}$ ac.
6. 168 men.

Ex. LXXXV, pp. 82, 83, 84.

1. £4 0*s.* $4\frac{2}{7}$*d.*
2. £1603 10*s.* $3\frac{2}{3}$*d.*
3. 243 bushels.
4. £24 8*s.* $7\frac{221}{613}$*d.*
5. 15 c. 0 qrs. $12\frac{2}{3}$ lbs.
6. 136 galls.
7. £6 9*s.* $5\frac{1}{3}$*d.*
8. 40 c. 0 qrs. 8 lbs.
9. £2 12*s.* $8\frac{1129}{3208}$*d.*
10. £6 5*s.*
11. 38 oz. 3 dwt. $15\frac{3}{11}$ grs.
12. £12 10 $3\frac{9}{32}$*d.*
13. £46 16*s.* 10*d.*
14. £118 12*s.* 6*d.*
15. £685 10*s.* $6\frac{2}{3}$*d.*
16. £406 1*s.* $4\frac{16}{25}$*d.*
17. £1083 15*s.*
18. £16 17*s.* $9\frac{3}{4}$*d.*
19. £580.
20. £18 0*s.* 4*d.*
21. £34 7*s.* $8\frac{3}{4}$*d.*
22. £370.
23. £116.
24. 8*d.*
25. £22 9*s.* $9\frac{3}{4}$*d.*
26. £60.
27. 6*s.* 8*d.*
28. £1548 13*s.* $10\frac{22}{23}$*d.*
29. £2073 12*s.* $8\frac{8}{11}$*d.*
30. 4*s.* 4*d.*
31. £103 6*s.* 3*d.*
32. £42 15*s.* 8*d.*
33. £2889 1*s.* $1\frac{31}{53}$*d.*

Ex. LXXXVI, p. 85.

1. £12 6*s.* $5\frac{38}{41}$*d.*
2. £9600.
3. 9*s.* $8\frac{277}{448}$*d.*
4. £1 1*s.* $10\frac{1}{2}$*d.*
5. £1 5*s.*
6. £198 5*s.*

Ex. LXXXVII, p. 87.

1. 126 men.
2. £15 12*s.* 6*d.*
3. 6 men.
4. 18 cows.
5. $3\frac{159}{160}$ miles.
6. $73\frac{1}{3}$ lbs.
7. 8 men.
8. 63 $\frac{9}{25}$ reams.
9. £47 3*s.* $5\frac{67}{118}$*d.*
10. £62 11*s.* $8\frac{37}{55}$*d.*
11. 45 men.
12. 17 horses.
13. $13\frac{1}{3}$ dys.
14. $7\frac{277}{299}$ dys.
15. $7\frac{1}{3}$ hrs.
16. 3 men.
17. $14\frac{1630}{3763}$ miles.
18. 160 men.
19. $14\frac{4}{13}$ dys.
20. 6 men.

MISCELLANEOUS EXAMPLES (LXXVI—LXXXVII), pp. 88, 89, 90.

1. £390.
2. $4\frac{11}{31}$ months.
3. £24 4*s.* $0\frac{18}{79}$*d.*
4. 13*s.* 4*d.*
5. £455.
6. £1 9*s.* $8\frac{5}{8}$*d.*
7. £272.
8. £4 4*s.* $8\frac{16}{19}$.
9. £112 3*s.* $8\frac{4}{19}$*d.*
10. £1 $3\frac{1}{25}$*s.*
11. £561 8*s.* 2*d.*
12. £16 10*s.* $5\frac{55}{61}$*d.*
13. £12 11*s.* $9\frac{3}{4}$*d.*
14. 168 ells.
15. 299 c. 2 qrs. $23\frac{43}{171}$ lbs.
16. 7*s.* 6*d.*
17. £2 $9\frac{1}{71}$*s.*
18. $20\frac{4}{13}$ dys.
19. £90 9*s.* 6*d.*
20. £1 12*s.* 6*d.*
21. £22 16*s.* $6\frac{6}{31}$*d.*
22. 4*s.* $6\frac{3}{7}$*d.*
23. £52 17*s.* 6*d.*
24. £3 7*s.* $3\frac{21}{61}$*d.*
25. £66 6*s.* $4\frac{7}{13}$*d.*
26. $8\frac{8}{119}$*d.*
27. £1619 0*s.* $11\frac{2}{7}$*d.*
28. £234 4*s.* 2*d.*
29. £14175 8*s.* $4\frac{2}{3}$*d.*
30. 6*s.* 8*d.*
31. £19 18*s.* $5\frac{1}{4}$*d.*
32. £422 13*s.* $9\frac{3}{4}$*d.*
33. £35 18*s.* $11\frac{1}{2}$*d.*
34. £33 15*s.*
35. £401 5*s.* $1\frac{83}{160}$*d.*
36. £228 6*s.* $5\frac{29}{32}$*d.*
37. £294 17*s.* $5\frac{3}{13}$*d.*
38. 6*d.*
39. £16316 1*s.* $7\frac{1}{2}$*d.*
40. £94 14*s.* $6\frac{6}{11}$*d.*

Ex. LXXXVIII, p. 92.

1. £70.
2. £184.
3. £1586.
4. £2546 6*s.* $10\frac{1}{2}$*d.*
5. £27 5*s.* 6*d.*
6. £88 0*s.* $11\frac{11}{20}$*d.*
7. £6 15*s.* $9\frac{9}{20}$*d.*
8. £553 12*s.* $10\frac{1}{80}$*d.*
9. £70 10*s.* $2\frac{3}{5}$*d.*
10. £21 2*s.* 6*d.*

Ex. LXXXIX, p. 92.

1. £59 15*s.* 10*d.*
2. £89 14*s.*
3. £315 4*s.* $7\frac{1}{4}$*d.*
4. £1600 9*s.* $7\frac{1}{365}$*d.*
5. £39 9*s.* $0\frac{36}{73}$*d.*
6. £386 10*s.* $1\frac{47}{73}$*d.*
7. £11 13*s.* $0\frac{12}{73}$*d.*
8. £267 5*s.* $7\frac{1}{2}$*d.*

Ex. XC, p. 93.

1. £28 4*s.* 3·168*d.*
2. £23 12*s.* 10·5*d.*
3. 19*s.* 1·665*d.*
4. £43 15*s.* 7·3056*d.*
5. £105.
6. 3*s.* $1\frac{1}{2}$*d.*

Ex. XCI, p. 94.

1. 8 yrs.
2. 3 yrs.
3. 7 yrs.
4. 9 yrs.

Ex. XCII, p. 95.

1. £1125. 2. £2275. 3. £850. 4. £1000.

Ex. XCIII, p. 95.

1. 3 p. c. 2. $3\frac{1}{7}$ p. c. 3. 5 p. c. 4. 4 p. c.

Ex. XCIV, pp. 96, 97.

1. £2 11*s*. $3\frac{33}{50}$*d*. 2. £8 9*s*. $10\frac{2}{25}$*d*. 3. £3 11*s*. $6\frac{9}{10}$*d*.
4. £7 17*s*. $7\frac{1279}{1460}$*d*. 5. £4 10*s*. $5\frac{101}{125}$*d*. 6. £6 18*s*. $10\frac{11}{25}$*d*.
7. £8 19*s*. $2\frac{3}{5}$*d*. 8. £7 0*s*. $3\frac{21}{25}$*d*.

Ex. XCV, p. 97.

1. £307 0*s*. $4\frac{4}{19}$*d*. 2. £774 9*s*. $8\frac{436}{439}$*d*. 3. £553 16 $7\frac{7}{47}$*d*.
4. £50 19*s*. $9\frac{3}{11}$*d*.

Ex. XCVI, p. 98.

1. £5 19*s*. 2*d*. 2. £23 15*s*. 3. £562 10*s*.
4. £7 2*s*. 6*d*. 5. £153 18*s*. $6\frac{3}{4}$*d*. 6. £39 16*s*. $10\frac{1}{2}$*d*.
7. £1 11*s*. 4·65*d*. 8. £1 2*s*. 6·135*d*. 9. £50.

Ex. XCVII, pp. 100, 102.

1. £4750. 2. £55 11*s*. $1\frac{1}{3}$*d*. 3. £350. 4. £5330.
5. £60. 6. £4250. 7. £3612 10*s*. 8. £4200.
9. £8160. 10. £5000. 11. £1250. 12. £3240.
13. £18,000. 14. £$96\frac{16}{19}$. 15. £14. 16. £312 10*s*.
17. 4 p. c. 18. £$63\frac{9}{19}$. 19. 4 p. c.

Ex. XCVIII, p. 104.

1. 9*s*. $11\frac{7}{10}$*d*. 2. 10*s*. $10\frac{10}{11}$*d*. 3. £1 12*s*. $7\frac{7}{23}$*d*.
4. 20 p. c. 5. £2 1*s*. $11\frac{71}{143}$*d*. 6. $14\frac{2}{7}$.
7. £3 15*s*. $6\frac{2}{11}$*d*., $8\frac{17}{176}$*d*. 8. £1 10*s*. 8*d*. 9. $8\frac{186}{333}$.
10. £17 3*s*. $4\frac{1}{2}$*d*. 11. $5\frac{17}{20}$. 12. $17\frac{1}{27}$.
13. 1*s*. $0\frac{21}{640}$*d*. 14. $31\frac{1}{4}$. 15. $8\frac{2}{5}$*d*., $7\frac{3}{5}$*d*.
16. $12\frac{1}{2}$. 17. 2*s*. $7\frac{47}{103}$*d*. 18. 10*s*. $0\frac{71}{2000}$*d*.

Ex. XCIX, p. 106.

1. $1996\frac{7}{8}$, $3328\frac{1}{8}$; $3328\frac{1}{8}$, $1996\frac{7}{8}$. 2. £500.
3. 7 cwt. 2 qrs. $22\frac{1}{2}$ lbs., 3 qrs. $19\frac{1}{2}$ lbs.
4. £16 13*s*. 4*d*., £33 6*s*. 8*d*., £50. 5. £54 3*s*. 4*d*. 6. £2 8*s*.

Ex. C, p. 108.

1. 2741. 2. 3005. 3. 6·25. 4. 97·8.
5. ·009456. 6. ·00950, &c. 7. $2\frac{3}{4}$. 8. $1\frac{2}{13}$.
9. $21\frac{5}{9}$. 10. $4\frac{4}{7}$.

Ex. CI, p. 110.

1. 13. 2. 125. 3. 74. 4. 809. 5. 279.
6. ·0015. 7. 43·87. 8. $\frac{7}{9}$. 9. $3\frac{1}{7}$. 10. $4\frac{1}{9}$.

MISCELLANEOUS EXAMPLES (LXXXIX—CII), pp. 110, 111, 112.

1. £175 10*s.* $10\frac{11}{160}$*d.* 2. £42 15*s.* $0\frac{61}{80}$*d.* 3. £49 3*s.* $10\frac{181}{383}$*d.*
4. £51 6*s.* $3\frac{3}{8}$*d.* 5. 20 yrs. 6. £469 7*s.* 6*d.*
7. $1\frac{2}{3}$. 8. £540 3*s.* $8\frac{7}{40}$*d.* 9. £1 7*s.* $7\frac{19}{20}$*d.*
10. £500. 11. £525. 12. £2 16*s.* $7\frac{1}{20}$*d.*
13. £1531 17*s.* 6*d.* 14. £53 12*s.* $3\frac{3}{4}$*d.* 15. £$6785\frac{5}{7}$.
16. £10290. 17. £$37\frac{1}{27}$. 18. 20 p. c.
19. £3 8*s.* $6\frac{3}{20}$*d.* 20. £2 1*s.* $1\frac{7}{20}$*d.* 21. 1*s.* $0\frac{297}{320}$*d.*
22. £83 6*s.* 8*d.*; £166 13*s.* 4*d.*; £250. 23. ·577 &c.; ·707, &c.
24. $12\frac{43}{80}$ yrs. 25. £64 3*s.* $0\frac{21}{400}$*d.* 26. £$7307\frac{1}{7}$.
27. £3 18*s.* $11\frac{7}{19}$*d.* 28. 4 p. c. at 80. 29. £1 4*s.* $0\frac{108}{1145}$*d.*
30. £465 5*s.* $3\frac{603}{619}$*d.* 31. 2·24, &c.; ·881, &c.; 2·07, &c.
32. £13 1*s.* $3\frac{39}{50}$*d.* 33. £1295 18*s.* $10\frac{47}{1825}$*d.* 34. $11\frac{1}{9}$ yrs.
35. $21\frac{7}{33}$ p. c. 36. $12441\frac{3}{17}$; $435\frac{15}{34}$. 37. 10*s.* $3\frac{741}{1633}$*d.*
38. $6\frac{2}{3}$ p. c. 39. 8*s.* $4\frac{1}{10}$*d.* 40. £538 7*s.* $2\frac{13}{20}$*d.*
41. £$14285\frac{5}{7}$. 42. $4\frac{47}{82}$*d.* 43. $1\frac{64}{191}$*d.*; $12\frac{448}{671}$ p.c.
44. 41·53, &c.; 4·12, &c. 45. ·003. 46. £1 7*s.* $10\frac{17}{21}$*d.*
47. £1 6*s.* 3*d.* 48. 4 p. c. at $93\frac{7}{8}$. 49. £671.
50. $7\frac{4}{26}$*d.*

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